Multi-Slit Interference and Discrimination of Quantum States

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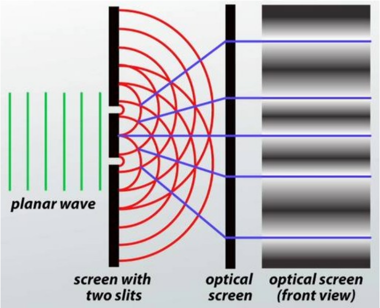
Outline

- Multi-slit experiment
- Quantum Coherence
  1. Visibility measure using quantum coherence
  2. Path predictability measure
  3. Which way information (Path distinguishability).
- Quantum Entanglement
  1. General concurrence Measure of Entanglement of the system and it's connection with the I concurrence.
  2. A triality relation among the generalizeed predictibility, coherence and the I concurrence.

Single particle interference



1. Young's double slit experiment, performed with a single photon, exhibits interference phenomena.
2. Particle nature is observed as a single localized pulse at a specific position in the path of the particle.
3. Wave nature of the particle is manifested by the interference pattern.

The role of Quantum Coherence in Double Slits exp.

In double slit experiment, superposition of two waves emitted by two slits. If the functions ψ_1 and ψ_2, which denote the waves reaching the screen emitted respectively by slits 1 and 2, represent two physically possible states of the system, then the linear superposition,

$$\psi = \psi_1 + \psi_2. \tag{1}$$

- The intensity, displayed on the screen by opening only 1st slit is $|\psi_1|^2$.
- The intensity, displayed on the screen by opening only 2nd slit is $|\psi_2|^2$.
- When the both slit is opened the intensity becomes,

$$|\psi|^2 = |\psi_1|^2 + |\psi_2|^2 + \psi_1^*\psi_2 + \psi_1\psi_2^* \tag{2}$$

The last two terms considered as off-diagonal term, give arises the *Quantum Coherence* and responsible for constructing the interference pattern.

Density matrix formalism of quantum state

Quantum system whose state is not completely known, more precisely a quantum system is one of a number of states $|\psi_i\rangle$ with respective probabilities p_i, it forms an ensemble of pure states. The density operator for a system can be defined in following fashion,

$$\rho = \sum_i p_i |\psi_i\rangle \langle\psi_i| \tag{3}$$

Criteria of representing a valid quantum state by density matrix,

- Density matrix should be Hermitian $\implies \rho^\dagger = \rho$
- Density matrix should be normalized$\implies Tr[\rho] = 1$

Density matrix of a two qubit state is,

$$\rho = 1/2 \begin{pmatrix} 1+z & x-iy \\ x+iy & 1-z \end{pmatrix} \tag{4}$$

The eigen values lie in the diagonal elements, and the off-diagonal elements are responsible for coherence.

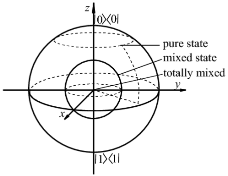
The purity condition,

- For pure state, $Tr[\rho^2] = 1$ Only Single state vector involves.
- For mixed state, $Tr[\rho^2] \leq 1$ Multiple state vectors involve.

A general single qubit mixed state is written in this form,

$$\rho = \frac{1}{2}(\mathcal{I} + \vec{n}.\vec{\sigma}) \qquad 0 \leq |\vec{n}| \leq 1. \tag{5}$$

The above eqt. is also considered as a state of a partially polarized light.



- When $|\vec{n}| = 1$, the state lies on the surface of the Bloch sphere and the state becomes pure state.
- For $|\vec{n}| < 1$, the state lies inside the sphere and the state can be considered as a mixed state.
- $\vec{n} = 0$ implies the state is in the center of the sphere, and the state is called maximally mixed state.

Motivation of our study

- Wootters and Zurek[1] were the first to quantitatively analyze the phenomenon of complementarity or wave-particle duality.
- Greenberger and Yasin[2] analyzed the wave-particle duality with the assumption that unequal beams in a two-path interferometer allows for predicting, to a certain degree, which of the two paths the quanton followed and established the duality relation: $P^2 + V^2 \leq 1$ involving a path predictability P and an interference visibility, $V = (I_{max} - I_{min})/(I_{max} + I_{min})$, with I_{max}, I_{min} being the maximum and minimum intensities in a region on the screen.
- Englert[3] studied two-path interferometer in the presence of a path-detector and he arrived at a duality relation $D^2 + V^2 \leq 1$, involving a *path distinguishability* D and the fringe visibility V.
- We investigated the issue of complementarity in **multi-path interference** and showed that a robust entanglement measure, namely **I concurrence**, plays an essential role in it.

[1] William K. Wootters and Wojciech H. Zurek, Phys. Rev. D **19**, 473 (1979).
[2] Daniel M. Greenberger, Allaine Yasin, Phys. Lett. A **128**, 391 (1988).
[3] Berthold-Georg Englert, Phys. Rev. Lett. **77**, 2154 (1996)

PHYSICAL REVIEW LETTERS

Controlled Decoherence in Multiple Beam Ramsey Interference

Michael Mei and Martin Weitz

Phys. Rev. Lett. **86**, 559 – Published 22 January 2001

ABSTRACT

We have scattered photons from an interfering path of a multiple beam Ramsey interference experiment realized with a cesium atomic beam. It is demonstrated that in multiple beam interference the decoherence from photon scattering cannot only lead to a decrease but, under certain conditions, also to an increase of the Michelson fringe contrast. In all cases, the atomic quantum state loses information with photon scattering, as "which-path" information is carried away by the photon field. We outline an approach to quantify this which-path information from observed fringe signals, which allows for an appropriate measure of decoherence in multiple path interference.

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covering atomic, molecular, and optical physics and quantum information

Coherence, path predictability, and I concurrence: A triality

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ABSTRACT

It is well known that fringe contrast is not a good quantifier of the wave nature of a quanton in multipath interference. An alternative interference visibility, based on the Hilbert-Schmidt coherence is introduced. It is demonstrated that this visibility is a good quantifier of wave nature and can be experimentally measured. A generalized path predictability is introduced, which reduces to the predictability of Greenberger and Yasin for the case of two paths. In a multipath, which-way interference experiment, the new visibility, the predictability, and the I concurrence (quantifying the entanglement between the quanton and the path detector) are shown to follow a tight *triality* relation. It *quantifies* the essential role that entanglement plays in multipath quantum complementarity.

Visibility Measure

- Greenberger and Yasin have shown that the fringe contrast(visibility, $V = (I_{max} - I_{min})/(I_{max} + I_{min})$) is not a good quantifier of the wave nature of a quanton in multipath interferometer.
- To address the above issue, we introduce a generalized visibility for a quanton passing through a multipath interferometer, we use the Hilbert-Schmidt coherence,

$$\nu^2 = \frac{n}{n-1} \sum_{i \neq j} |\rho_{ij}|^2 = \mathcal{C}_{HS} \tag{6}$$

This visibility is a good quantifier of the wave nature of a quanton and that can be experimentally measured and it also helps in detecting the path information.

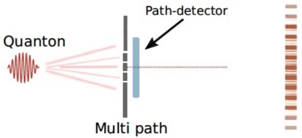
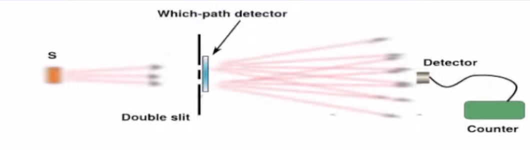


FIG. 1. A schematic representation of a multipath interference experiment, with which-way detection

Consider a quanton passing through n-path interferometer, the corresponding state can be described in the density matrix formalism,

$$\rho = \sum_{j=1}^{n} \sum_{k=1}^{n} \rho_{jk} \left|\psi_j\right\rangle \left\langle\psi_k\right| e^{i(\theta_j - \theta_k)} \tag{7}$$

We have taken in counter the possibility that the phase in the i^{th} beam may be shifted by θ_i in order to analyze the experimental scenario and the ρ_{jk} are assumed to be real.



The particle goes to upper slit $\longrightarrow$ path-detector state $|d_1\rangle$.
The particle goes to lower slit $\longrightarrow$ path-detector state $|d_2\rangle$.

- The combined entangled state of the quanton and the path detector in multi-path interferometer,

$$\rho' = \sum_{j=1}^{n}\sum_{k=1}^{n} \rho_{jk}\,|\psi_j\rangle\langle\psi_k|\,e^{i(\theta_j-\theta_k)} \otimes |d_j\rangle\langle d_K| \tag{8}$$

where $\{|d_i\rangle\}$ are the states of the path detector, they are normalized, but not necessarily mutually orthogonal.

- The reduced density operator of the quanton is obtained by tracing over the path-detector states:

$$\rho'_r = \sum_{j=1}^{n}\sum_{k=1}^{n} \rho_{jk}\,|\psi_j\rangle\langle\psi_k|\,e^{i(\theta_j-\theta_k)}\langle d_j|d_K\rangle \tag{9}$$

The path-detector is assumed to be able to only tell if the quanton passed through path l, it cannot discriminate between any other path.
The visibility can be obtained for this state of the quanton,

$$\nu'^2 = \frac{n}{n-1} \sum_{i \neq j} |\rho_{ij}|^2 |\langle d_k | d_j \rangle|^2 \tag{10}$$

Since $|\langle d_k | d_j \rangle| = 1$, it can be easily concluded $\nu'^2 \leq \nu^2$ which implies that **the visibility will never increase if one tries to find out whether the quanton passed through the path l or not.**

- **How to measure this visibility in an interference experiment?**

We can assume that beam of each quanton is split into new channels (as happens in a Mach-Zehnder interferometer) whose states can be represented by $|\xi_i\rangle$. We are assuming the all beams have equal overlap with the particular output channel $|\xi_i\rangle$, which implies $\langle \xi_i | \psi_K \rangle = A_i$ and it will remain same for all k. The probability of finding the quanton in the i^{th} output channel (not the i^{th} path) is following,

$$I = |A_i|^2 \left[1 + \sum_{j \neq k} |\rho_{jk}| \cos\left(\theta_j - \theta_k\right) \right] \tag{11}$$

The probability term can also be interpreted as the intensity at a position on the screen and it depends upon the phase of the path.

From the probability term we can obtain the following term,

$$\langle (\Delta I)^2 \rangle_\theta = \frac{1}{n^2} \sum_{i \neq j} |\rho_{ij}|^2 \tag{12}$$

Where $\Delta I = I - \langle I \rangle_\theta$ and $\langle I \rangle_\theta$ denotes the average over all phases. The new visibility now we can write in term of the intensity,

$$\nu^2 = \frac{n^3}{n-1} \langle (\Delta I)^2 \rangle_\theta \tag{13}$$

which indicates the visibility can be measured if one can design an experimental arrangement to vary the phases of all paths explicitly and then average over them.

This is the method of experimentally obtaining the new visibility.

Predictability Measure

- **What is predictability measure?**
  Predictability is defined as the difference of the diagonal elements, and encodes the difference in probabilities of the outcomes for this two level system. In case of two slit interference with single photon, it is the difference of probabilities of going through either slit.

A generalized predictability can be defined in the following way,

$$\mathcal{P}^2 = \sum_{i=1}^{n} \rho_{ii}^2 - \frac{1}{n-1} \sum_{i \neq j} \rho_{ii}\rho_{jj} = 1 - \frac{n}{n-1} \sum_{i \neq j} \rho_{ii}\rho_{jj} \tag{14}$$

Where ρ is the density matrix of the quanton and n is the dimension of the density matrix or the number of paths. It follows few criteria,

- $\mathcal{P}$ is a polynomial function of ρ_{ii} and it is continuous function involving only the diagonal elements.
- If the quanton's path is known with certainty, i.e., $\rho_{ii} = 1$ and $\rho_{jj} = 0$ which indicates the physical phenomenon where the only i^{th} slit is open and predictability reaches its global maxima, i.e. $\mathcal{P} = 1$.
- When the quanton's probability of going through all paths are equally likely, i.e., $\rho_{ii} = \frac{1}{n}$, that time the predictability $\mathcal{P}$ becomes *zero*.

Our proposed generalized predictability can be reduced into two-beam interferometer case ($n = 2$),

$$\mathcal{P} = |\rho_{11} - \rho_{22}| \tag{15}$$

The multi-path predictability can also be written as root mean square of the two-path predictabilities from all path pairs:

$$\mathcal{P} = \frac{1}{n-1}\sum_{pairs}(\rho_{ii} + \rho_{jj})^2 P_{ij}^2 \tag{16}$$

Where, P_{ij} is the path predictability for the i^{th} and j^{th} path. Our n-path predictability can also be expressed in term of the conventional two-slit predictability for path pair has not been recognized earlier.

Path Distinguishability

one can talk of path distinguishability in the context of experimentally determining through which slit did the quanton pass.

- A multi-path distinguishability has been proposed earlier as,

$$\mathcal{D}^2 = 1 - \left(\frac{1}{n-1} \sum_{i \neq j} \sqrt{\rho_{ii}\rho_{jj}} |\langle d_i | d_j \rangle| \right)^2 \tag{17}$$

- We define a new path-distinguishability $\mathcal{D}$ in a different way in order to being a more reliable path quantifier.

$$\mathcal{D}^2 = 1 - \frac{n}{n-1} \sum_{i \neq j} \rho_{ii}\rho_{jj} |\langle d_i | d_j \rangle|^2 \tag{18}$$

Our proposed distinguishability measure satisfies the all following basic criteria of a ideal path quantifier,

- When the detector sate $\{|d_i\rangle\}$ from an orthonormal basis, i.e., $\langle d_i|d_j\rangle = \delta_{ij}$, the path distinguishability is maximum ($\mathcal{D} = 1$), since in this case all the path can be unambiguously discriminated.
- When all the detector states are parallel, i.e., state of quanton and the detector is separable, and probability through all the slits are equal, ie., $\rho_{ii} = \frac{1}{n}$, for all i, then the path distinguishability is minimum ($\mathcal{D} = 0$).
- Whenever the detector states $|d_i\rangle$ and $|d_j\rangle$ are made more orthogonal, i.e., with an increment in $|\langle d_i|d_j\rangle|$, this distinguishability increases.
- Distinguishability and visibility cannot both increases simultaneously under any operation, which is evident through the fact $\mathcal{D}^2 + \nu^2 = 1$

- When the all path are equally probable, the *meansquare* of the distinguishability, from all pairs of paths, is given by

$$\frac{2}{n(n-1)}\sum_{pairs}\mathcal{D}_{ij}^2 = 1 - \frac{n}{n-1}\sum_{i\neq j}\rho_{ii}\rho_{jj}|\langle d_i|d_j\rangle|^2 = \mathcal{D}^2\ . \tag{19}$$

  With the help of above equation, the distinguishability too can be measured experimentally as the *root mean square* of conventional distinguishabilities from all path pairs.

- The paths are not equally probable, the distinguishability can be measured through the conventional two-slit distinguishability from path pairs D_{ij} and the path probabilities ρ_{ii} as follows,

$$\mathcal{D}^2 = \frac{n}{2(n-1)}\sum_{pairs}(\rho_{ii}+\rho_{jj})^2 D_{ij}^2. \tag{20}$$

Complementarity

- Let's consider a quanton passing through an n-path interferometer.
- Using the definitions of visibility in terms of Hilbert-Schmidt coherence (Eqn. 6) and predictability (Eqn. 14) we obtain the duality relation

$$\mathcal{P}^2 + \mathcal{V}^2 = 1 - \frac{n}{n-1} \sum_{i \neq j} (\rho_{ii}\rho_{jj} - |\rho_{ij}|^2) \tag{21}$$

- If the state of quanton is pure ($\rho = \sum_{i,j} c_i c_j^* \left|\psi_i\right\rangle \left\langle\psi_j\right|, \ \{c_i\} \in \mathbb{C}$), duality relation saturates.
- For mixed state of quanton with elements given by $\rho_{ij} = \sum_\alpha p^\alpha c_i^\alpha c_j^{\alpha *}$,

$$(\rho_{ii}\rho_{jj} - |\rho_{ij}|^2) = \sum_\alpha |c^\alpha|^2 \sum_\beta |c^\beta|^2 - |c_i^\alpha c_j^{\beta *}|^2 \geq 0 \ . \tag{22}$$

  Above inequality follows from the Cauchy-Schwarz inequality.
- So, in general we have the following n-path duality relation (Dürr *et al.* [4]):

$$\mathcal{P}^2 + \mathcal{V}^2 \leq 1 \ . \tag{23}$$

[4] Phys. Rev. A **64**, 042113 (2001).

- Consider an interferometer with an added path-detecting device, such that the quanton going through the n paths is described by the state

$$|\Psi\rangle = \sum_{i=1}^{n} c_i |\psi_i\rangle |d_i\rangle \tag{24}$$

where $|\psi_i\rangle$ is the state of the quanton corresponding to its passing through the i^{th} path and $|d_i\rangle$ is the state of the path detector, corresponding to that possibility.

- $\{|d_i\rangle\}$ are assumed to be normalized but not necessarily orthogonal.
- The reduced density matrix of the quanton, obtained by tracing over the path-detector states, is described by (Eqn. 9) with $\{\theta_j\} = 0$ and $\rho_{ij} = c_i c_j^*$.
- Duality relation in this case is given by:

$$\mathcal{P}^2 + \mathcal{V}^2 = 1 - \frac{n}{n-1} \sum_{i \neq j} \rho_{r\,ii} \rho_{r\,jj} + \frac{n}{n-1} \sum_{i \neq j} |\rho_{r\,ij}| \tag{25}$$

where $\rho_{r\,ij} = c_i c_j^* \langle d_i | d_j \rangle$.

- We can define a normalized entanglement measure such that

$$\mathcal{E} = \frac{n}{n-1}\sum_{i \neq j}(\rho_{r\,ii}\rho_{r\,jj} - |\rho_{r\,ij}|^2) = \frac{n}{2(n-1)}E^2 \qquad (26)$$

where, E is the generalized concurrence and coincides with I concurrence when the bipartite state under consideration is a pure state.

- Eqn. (25) can then be written as

$$\mathcal{P}^2 + \mathcal{V}^2 = 1 - \mathcal{E}^2 \qquad (27)$$

or as a triality relation

$$\mathcal{P}^2 + \mathcal{V}^2 + \mathcal{E}^2 = 1\ . \qquad (28)$$

- Therefore, the generalized predictability, Hilbert-Schmidt coherence, and I concurrence obey a tight triality relation.
- The path-distinguishability measure (Eqn. 18), is related to the generalized predictability through I concurrence as follows:

$$\mathcal{D}^2 = \mathcal{P}^2 + \mathcal{E}^2\ . \qquad (29)$$

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Generalized concurrence measure for faithful quantification of multiparticle pure state entanglement using Lagrange's identity and wedge product

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Abstract Concurrence, introduced by Hill and Wootters (Phys Rev Lett 78:5022, 1997), provides an important measure of entanglement for a general pair of qubits that is faithful: strictly positive for entangled states and vanishing for all separable states. Such a measure captures the entire content of entanglement, providing necessary and sufficient conditions for separability. We present an extension of concurrence to multiparticle pure states in arbitrary dimensions by a new framework using the Lagrange's identity and wedge product representation of separability conditions, which coincides with the "I-concurrence" of Rungta et al. (Phys Rev A 64:042315, 2001) who proposed by extending Wootters's spin-flip operator to a so-called universal inverter superoperator. Our framework exposes an inherent geometry of entanglement and may be useful for the further extensions to mixed and continuous variable states.

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Non-destructive discrimination of Bell states by NMR using a single ancilla qubit

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Abstract
Discrimination of Bell states plays an important role in a number of quantum computational protocols such as teleportation and secret sharing. However, most of the protocols dealing with Bell state discrimination in the literature either involve performing correlated measurements or destroying the entanglement of the system. Here, we demonstrate an NMR-based experimental realization of a protocol for Bell state discrimination, following a scheme proposed by Gupta *et al* (quant-ph/0504183v1, 23 April 2005), which does not destroy the Bell state under consideration. Using the proposed protocol, one can deterministically distinguish the Bell states, without performing a measurement using the entangled basis. State discrimination is performed through two independent measurements on one ancilla qubit, which leaves the Bell states unchanged.

Bell state discrimination and error correction

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An ancilla-based single particle measurement scheme is illustrated for Bell state discrimination which does not lead to the collapse of the wave function. It enables the extraction of partial information, e.g., parity and phase independently, that generalizes to multi-particle entangled states of qubits, which also applied to qudits. It can be used for error correction in a quantum circuit architecture involving basic gates like Hadamard and Controlled-NOT. It helps unravel the syndrome operators and, what is more, in the multi-partite state that the individual constituents lack.

Thank You