CHALLENGING THE DIVIDE BETWEEN CLASSICAL DETERMINISM AND QUANTUM INDETERMINISM

Giulio Chiribella

QICI, School of Computing and Data Science, The University of Hong Kong

International Symposium on
Quantum Information and Quantum Communication
March 31-April 2, 2025, CQuERE, TCG CREST

QUANTUM VS CLASSICAL

Quantum theory is often contrasted with classical theory,
regarded as the golden standard of a theory that reflects our intuitions about reality:

a theory that describes a world of objects
that have definite properties,
independent of whether they are measured or not.

But is the gap between quantum and classical really as large as it seems at first sight?

THE ROLE OF PROBABILITIES

In the common lore, one of the key differences concerns the role of probabilities.

Textbook interpretation of quantum theory

Fundamentally probabilistic: the theory predicts the probabilities of measurement outcomes.

The probability of an outcome is the likelihood that that outcome occurs ***if the corresponding measurement is performed.***

Textbook interpretation of classical theory

Fundamentally deterministic: probabilities only reflect the ignorance of an agent.

The probability of a value is the likelihood that the quantity has that value, ***independently of whether it is measured or not.***

ALTERNATIVE INTERPRETATIONS OF CLASSICAL THEORY?

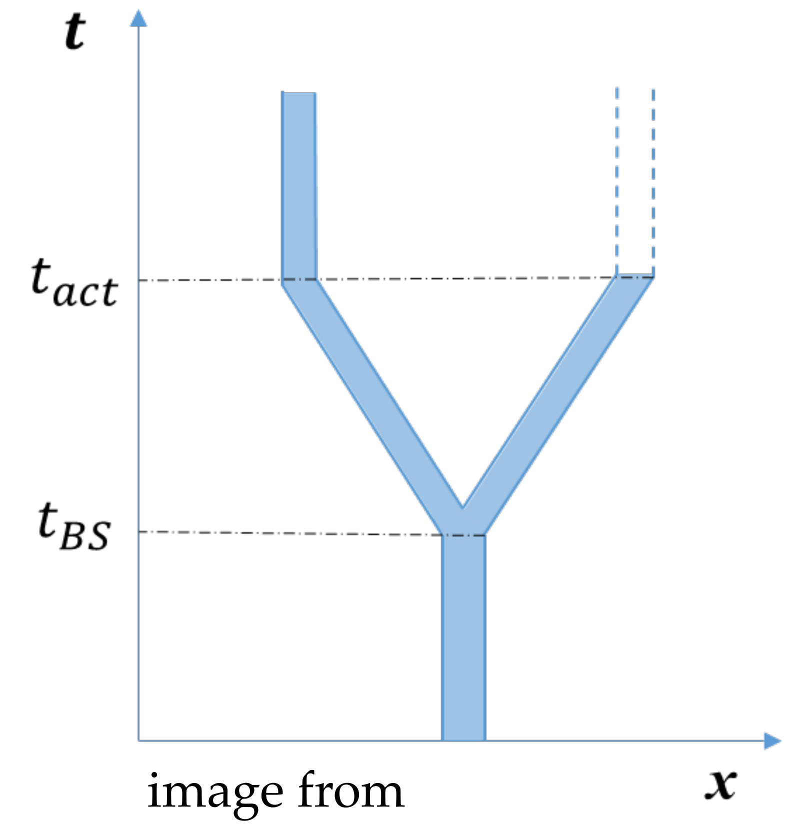


image from
Del Santo and Gisin,
arXiv:2409.10601

Recently, several works explored
alternative interpretations of classical theory.

In particular, Gisin and Del Santo propose
a variant of classical theory
where the values of classical quantities are not fully determined.

They argued for this interpretation on the ground that continuous quantities,
such as the position of a particle, may only be defined with a finite amount of precision.

N. Gisin, Erkenntnis 86, 1469 (2019).
F. Del Santo and N. Gisin, Phys. Rev. A 100, 062107 (2019).

THIS TALK

In this talk,
I will show that, in principle,
the traditional interpretation
of classical physics can be falsified
if classical systems coexist
with other types of physical systems.

Technically, I will show a **toy theory,**
which includes classical theory
as a subtheory,
describing a part of the world.

Agents who only observe that part
of the world can still believe that objects
properties prior to measurement.

Agents that observe a larger part of
the world can falsify this belief.

THE TOY THEORY

G. Chiribella, L. Giannelli, and C. M. Scandolo, Phys. Rev. Lett. 132, 190201 (2024)

WHAT DO YOU MEAN BY "THEORY"?

Here we will consider the framework of operational-probabilistic theories (OPTs).

An OPT

- describes a set of physical systems, closed under composition
  (if the theory describes systems A and B individually,
  it also describes the composite system AB)

- for every system, it specifies its possible states and the possible processes
  it can undergo

- it specifies a set of experiments that can be performed,
  and assigns probabilities to the experimental outcomes.

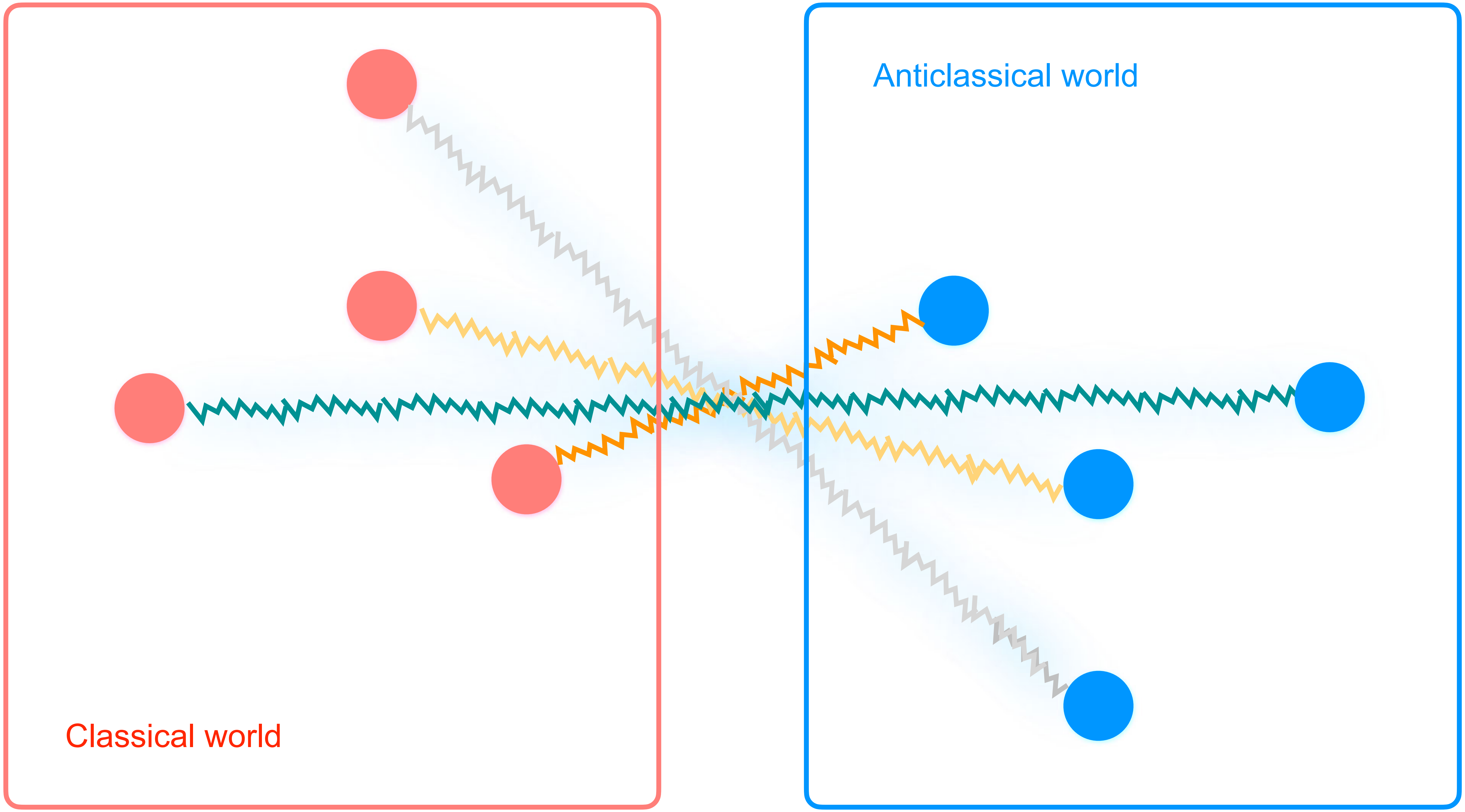
G. Chiribella, G. M. D'Ariano, and P. Perinotti, Phys. Rev. A 81, 062348 (2010)
L. Hardy, in Deep Beauty: Understanding the Quantum World Through Mathematical Innovation 409 (2011)
G. M. D'Ariano, G. Chiribella, and P. Perinotti, Quantum Theory from First Principles Cambridge University Press (2016)

BITS AND ANTI-BITS

In our toy theory, there are two basic types of systems:
bits and anti-bits.

Every other system is a composite system,
made of some number of bits, and some number of anti-bits.

Anti-bits have the same state space as bits,
but are in principle distinguishable from them,
in a similar way as quantum particles are distinguishable from their anti-particles.



A composite system made only of bits follows the rules of classical theory.
A composite system made only of anti-bits also follows the rules of classical theory.

Hybrid composite systems, containing both bits and anti-bits, give rise to non-classical phenomena.

(1,1) COMPOSITES

The simplest non-classical composite is made of 1 bit and 1 anti-bit.

To construct the composite, we use the Hilbert space framework
without however assuming quantum mechanics.

- The pure states are of the form $|\psi_{\mathrm{even}}\rangle = \alpha\,|0\rangle \otimes |0\rangle + \beta\,|1\rangle \otimes |1\rangle$
  or of the form $|\psi_{\mathrm{odd}}\rangle = \alpha'\,|0\rangle \otimes |1\rangle + \beta'\,|1\rangle \otimes |0\rangle$
  where $\alpha, \alpha', \beta, \beta'$ are complex amplitudes.

- The mixed states are mixtures of these pure states,
  and can be described by density matrices subject to a superselection rule on the parity.

- The measurements are represented by resolutions of the identity into positive operators that satisfy the parity superselection rule.

G. Chiribella and C. M. Scandolo, https://arxiv.org/abs/1608.04459 (2016)

LOCAL MEASUREMENTS

The ideal measurement on the bit is described by the resolution$\{ |0\rangle\langle 0| \otimes I, |1\rangle\langle 1| \otimes I\}$.
The ideal measurement on the anti-bit is described by $\{I \otimes |0\rangle\langle 0|, I \otimes |1\rangle\langle 1|\}$.
The other local measurements are noisy versions of the ideal measurements.

Measuring the anti-bit will collapse the bit into a mixture of the pure states $|0\rangle$ and $|1\rangle$ and *vice-versa:*
no superposition states for bits, no superposition states for anti-bits.

ENTANGLEMENT

The only product states of the bit/anti-bit composite are
$|0\rangle \otimes |0\rangle$, $|0\rangle \otimes |1\rangle$, $|1\rangle \otimes |0\rangle$, and $|1\rangle \otimes |1\rangle$
These states represent the situation in which the bit and the anti-bit have definite values.

All the other pure states are **entangled**, in Schrödinger's sense:
"maximal knowledge of a whole does not imply maximal knowledge of the parts."

…***BUT***

The composite of a bit and an anti-bit is not the same as the composite of two qubits!
The entanglement in the (1,1) composite

- cannot be detected by correlations between local measurements (violation of tomographic locality)

- does not give rise to the violation of Bell inequalities.

CONSISTENCY OF THE THEORY

The composition rule for systems made of m bits and n anti-bits is a bit more involved. I will omit it here.

Long story short: *we can set up appropriate composition rules that define a consistent OPT.*

Theorem (consistency of conditional states)

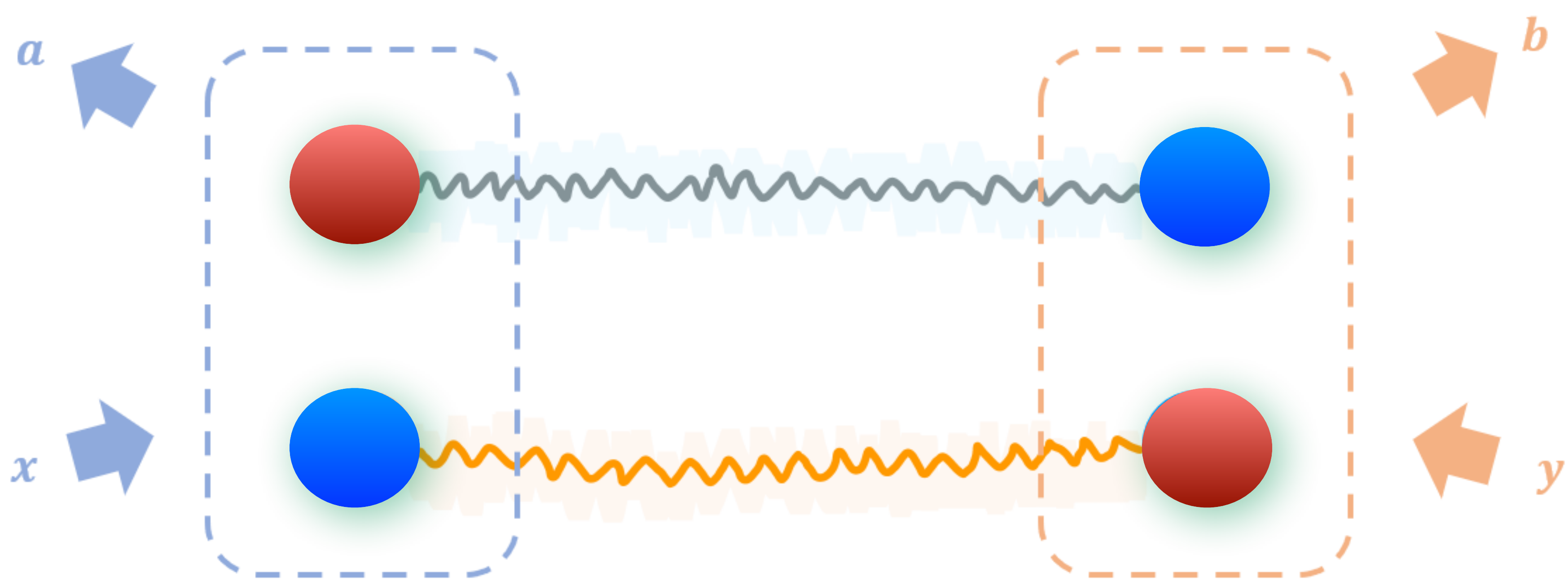
For every pair of (possibly composite) systems A and B,
an for every joint state of the composite system AB,
a local measurement on system A collapses system B to a valid state.

VIOLATION
OF
BELL INEQUALITIES
IN
THE TOY THEORY

ACTIVATION OF NONLOCALITY IN THE TOY THEORY

We have seen that the entanglement between a single bit and a single anti-bit cannot give rise to Bell inequality violations.

However, Bell inequality violations become observable if ***two identical copies*** of an arbitrary entangled pair are available.



Alice's laboratory

Bob's laboratory

ALICE'S BIT CANNOT BE PRE-DETERMINED

We find that the CHSH correlation can reach the maximum value $2\sqrt{2}$
and that this violation can be achieved by a setup where
one of Alice's setting corresponds to the ideal measurement of the value of the her bit.

In a world described by our toy theory,
the assumption that classical bits have pre-determined values
can (in principle) be experimentally falsified!

LIMITS ON CHSH VIOLATION WHEN ONE OF THE OUTCOMES IS PRE-DETERMINED

Consider an ontic model that satisfies a minimal locality condition and assigns a definite value to Alice's bit:

- the ontic model is specified by an ontic state λ, a probability distribution $p(\lambda)$, and a response function $q_{AB}(a, b \,|\, x, y, \lambda)$.
- the minimal locality condition is that the response function $q_{AB}(a, b \,|\, x, y, \lambda)$ is no-signaling for every possible ontic state λ
- the condition that Alice's bit is pre-determined amounts to the condition that there exists a value $a_* \in \{0,1\}$ such that $q_A(a_* \,|\, x = 0,\lambda) \geq 1 - \epsilon$ for some (small) $\epsilon > 0$

Theorem. For every ontic model satisfying these three conditions the CHSH correlation is bounded as $\text{CHSH} \leq 2(1 + 2\epsilon)$.

OUTLOOK

TAKE HOME MESSAGES

- Our toy includes classical theory as a subtheory
  describing a subset of physical systems.
  Classical systems can be entangled with other types of systems,
  giving rise to globally non-classical behaviors.

- In a world described by our toy theory, the violation of Bell inequalities
  can be used to falsify the assumption
  that the properties of classical systems are defined prior to measurement.

- Our toy theory suggests that issues with the interpretation of quantum mechanics
  may be deeper than the distinction between quantum and classical physics.
  The solution of these issues may require more radical steps,
  viz. many-world interpretations, QBism,…

- Open problem: include continuous classical systems in the toy theory.