Dynamical Many Body Localization in Floquet Spin Chains

From Simple Quantum Magnets to Discrete Time Crystals

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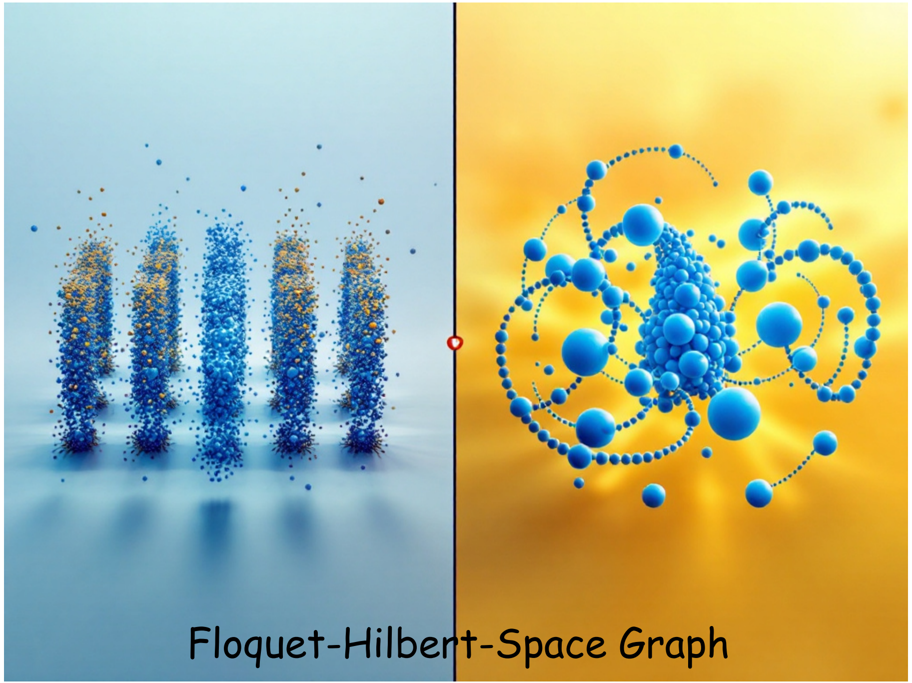
CQuERE, TCG CREST

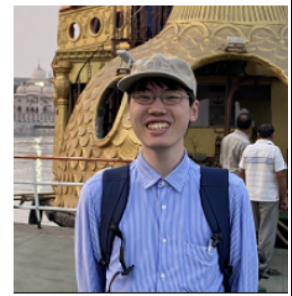
Introduction

- Many Body Localized systems retain memory of initial states, unlike thermalizing systems.
- Periodically Driven Quantum Systems can induce localization in energy space, even without disorder, leading to dynamical freezing and Dynamical Many Body Localization (DMBL).
- Quantum Interference plays a crucial role in DMBL, affecting ergodicity and energy absorption.
- MBL enables unique and exotic quantum orders like Floquet time crystals, protected by localization.



People

Collaborators:

Analabha Roy (BU) 	Takashi Mori (Keio U, Tokyo)  慶應義塾 Keio University	Akitada Sakurai (Okinawa Institute) OIST 
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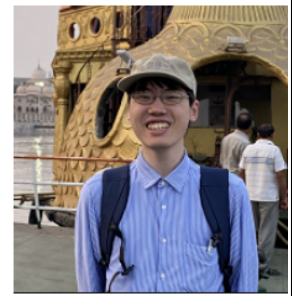
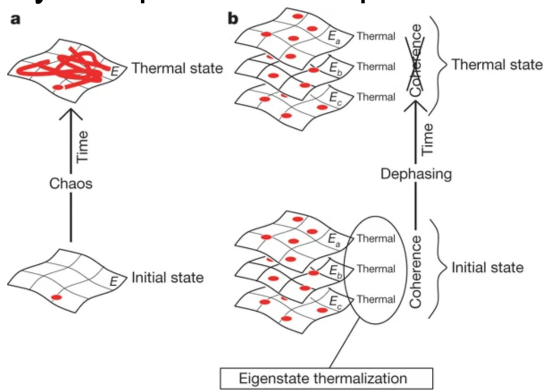
Analabha Roy (BU) 	Takashi Mori (Keio U, Tokyo)  慶應義塾 Keio University	Akitada Sakurai (Okinawa Institute) OIST 
Arnab Das (IACS, Kolkata) 	Mahbub Rahaman (BU) 	Wanna join? We're open to Ph.D. candidates I also work on Bio-Molecular Dynamics

Motivations: ETH-MBL

- Sufficiently complex closed quantum MB systems eventually thermalize dynamically (ETH)



$$\overline{A} \to \sum_\alpha |c_\alpha|^2 A_{\alpha\alpha} \approx \langle A\rangle_{mc} \sum_\alpha |c_\alpha|^2 = \langle A\rangle_{mc}$$

Rigol *et. al.* Nature (2008).

- However, quantum dynamics is supposed to preserve the memory of the initial conditions *i.e.* , *localized*.

$$|\psi(t)\rangle = \hat{U}(t)\,|\psi(0)\rangle \;\; , \;\; \partial_t \hat{U} = i\hat{H}\hat{U}$$

- Many Body Localization (MBL) is a phase of matter where this happens in local observables for long times.

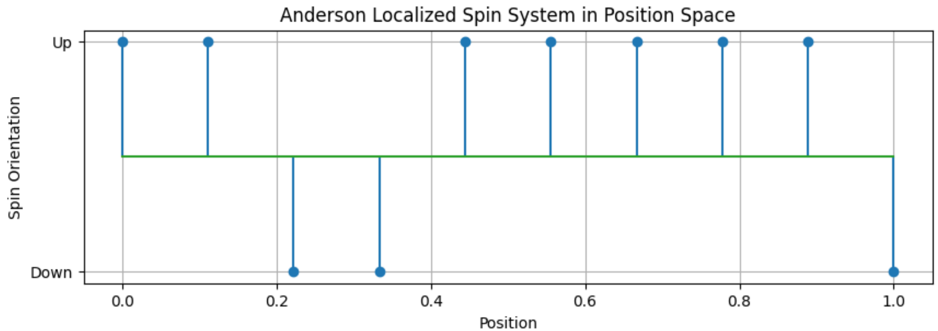
ETH (Thermal) MBL (Athermal)

Abanin *et. al.* Rev. Mod. Phys. 91, 021001

Motivations: ETH-MBL

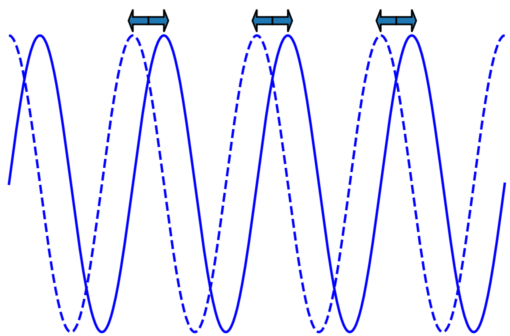
Background:
- [Anderson (1958), *Phys. Rev.* 109 (5): 1492]
- *Nandkishore and Huse (2015), arXiv:1404.0686*
- Morningstar et. al. (2022): arXiv: 2107.05642

- MBL uses interactions to build on Anderson Localization: Spatially-localized dynamics by disorder.
- Thus, the closed system keeps initial condition information, even under thermodynamic conditions.



Anderson localization in this context means that the spins are localized at certain sites due to disorder in the system

- In time periodic systems, MBL can be regulated with high-frequency drives (Floquet Engineering) Even without disorder! This is important, as disorder-induced MBL is prethermal.



$$\hat{H} \to \hat{H}(t) \quad \hat{H}(t + nT) = \hat{H}(t)$$

Oka and Kitamura, Ann. Rev. Cond. Matt. Phys. (2018).

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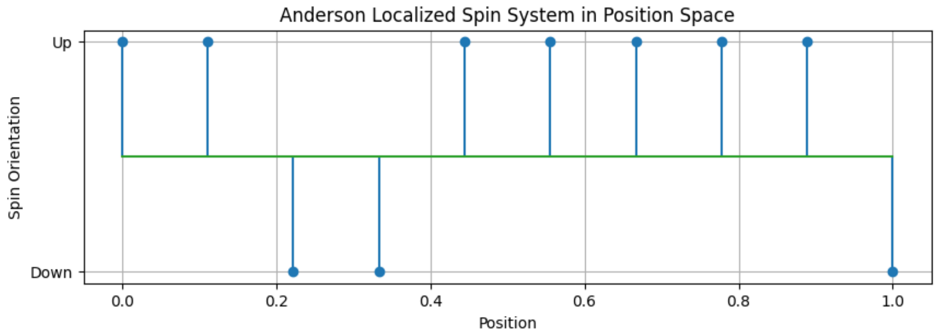

MBL talk on YouTube by David Huse

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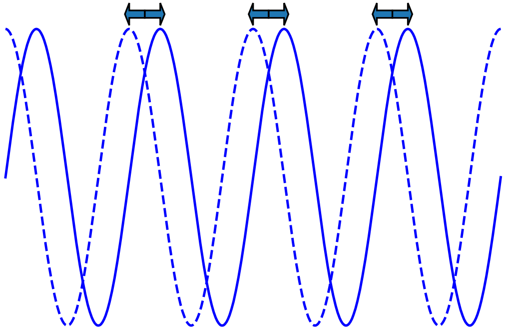

Floquet talk on YouTube by Takashi Oka

Paradigmatic many body example

Theory:

- E. Lieb, T. Schultz, and D. Mattis, *Ann. Phys 16:407 (1961)*
- Bigan Mbeng *et. al.* arXiv:2009.09208
- G. Mussardo- *Statistical Field Theory* Oxford (2020), ISBN:978-0198788102

Transverse Field Ising Model with harmonic drive

$$H(t) = -\frac{1}{2}\left[J\sum_i \sigma_i^x \sigma_{i+1}^x + h(t)\sum_i \sigma_i^z\right] \qquad h(t) = h\cos\omega t + h_0$$

$$\begin{aligned} \sigma_i^x &\equiv \left(b_i^\dagger + b_i\right) \\ \sigma_i^y &\equiv -i\left(b_i^\dagger - b_i\right) \\ \sigma_i^z &\equiv 2b_i^\dagger b_i + 1 \end{aligned} \longrightarrow b_j = e^{i\pi\sum_{i<j} c_i^\dagger c_i} c_j$$

Jordan Wigner Transformation

$$H(t) = \sum_{k,-k}\left(c_k, c_{-k}^\dagger\right)\begin{pmatrix} h(t)+f_k & \Delta_k \\ \Delta_k^* & -h(t)-f_k \end{pmatrix}\begin{pmatrix} c_k^\dagger \\ c_{-k} \end{pmatrix}$$

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Map to independent TLS systems

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Experiment:

- NMR (trifluoroiodoethylene) - Hegde *et. al.* arXiv:1307.8219(2013).

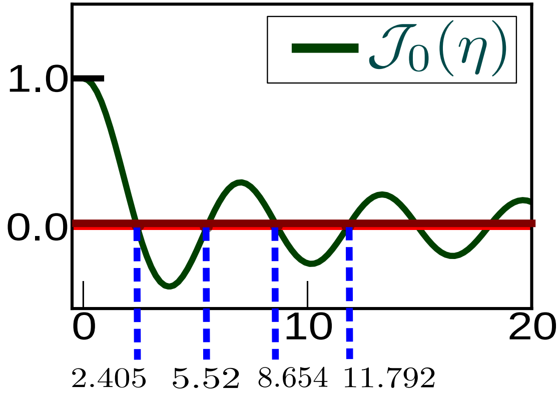
Transverse Field Ising Model with harmonic drive :

Unitary Transformation:

$$U(t) = \prod_k \exp\left\{ \left[\frac{ih}{\omega} \sin \omega t \right] \tau_z \right\}$$

$$H'(t) = \sum_{\substack{(k,-k) \\ \text{pairs}}} \left(c_k, c^\dagger_{-k} \right) \left\{ \tau_z f_k + 2\tau_x \Delta_k \sum_{n \geq 0} J_{2n}(\eta) \cos\left(2n\omega t\right) \right.$$

$$\left. - 2\tau^y \Delta_k \sum_{n \geq 0} J_{2n+1}(\eta) \sin\left[(2n+1)\, \omega t \right] \right\} \begin{pmatrix} c^\dagger_k \\ c_{-k} \end{pmatrix}$$



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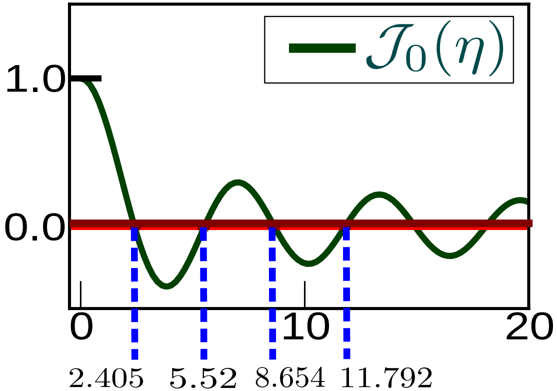

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$$f_k \ll \omega$$

zero-photon processes

Rotating Wave Approx. (RWA)

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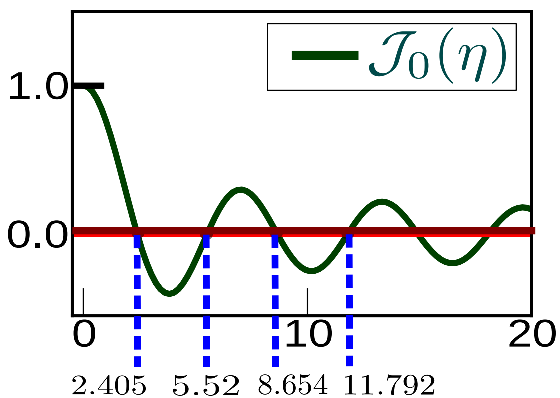

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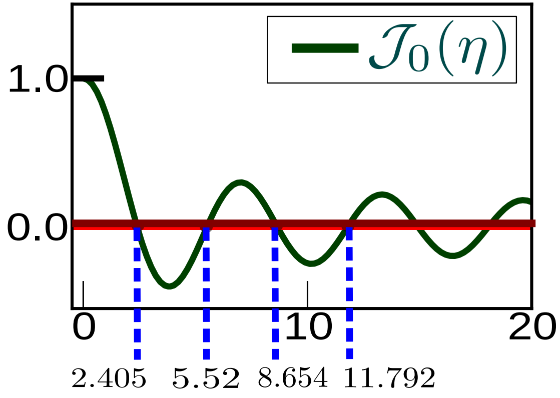

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Rotating Wave Approx. (RWA)

TFIM- Localization When Frozen

[M. Rahaman, T. Mori and A. Roy](arXiv:2308.03622)
arXiv:2308.03622

Localization, quantified by Inverse Participation (IPR) of Floquet Modes

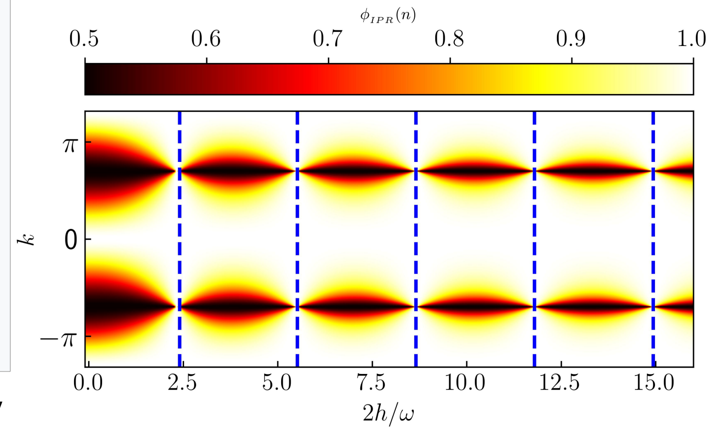
$$H(t) = \sum_k \left(c_k, c^\dagger_{-k}\right) \begin{pmatrix} h(t) + f_k & \Delta_k \\ \Delta^*_k & -h(t) - f_k \end{pmatrix} \begin{pmatrix} c^\dagger_k \\ c_{-k} \end{pmatrix},$$

$$\hat{U}(t) = e^{-i\hat{K}_F(t)}\, e^{-i\hat{H}_F t},$$

$$\hat{K}_F = \text{ Time-periodic micromotion, negligible for large-}\omega$$

$$\hat{H}_F = \text{ Floquet Hamiltonian}$$

$$\text{Eigs}\left[\hat{\mathrm{H}}_{\mathrm{F}}\right] \rightarrow \Omega_n \text{ Quasienergies, } |\phi^n\rangle = \prod_k |\phi^n_k\rangle \text{ Floquet Modes}$$



- IPR: Often defined as the integral of the square of density

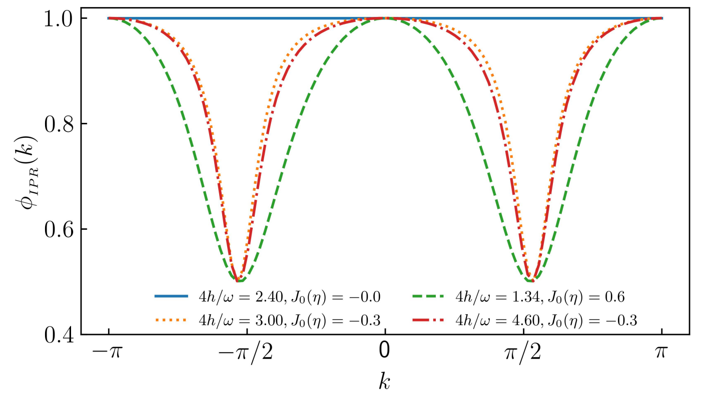
$$\mathrm{IPR} \equiv \int \mathrm{d}^3\boldsymbol{r}\, \rho^2(\boldsymbol{r})$$

- Reduced IPR:

$$\phi_{IPR} \equiv \sum_m |\langle m|\psi\rangle|^4.$$

- Reduced IPR: TFIM

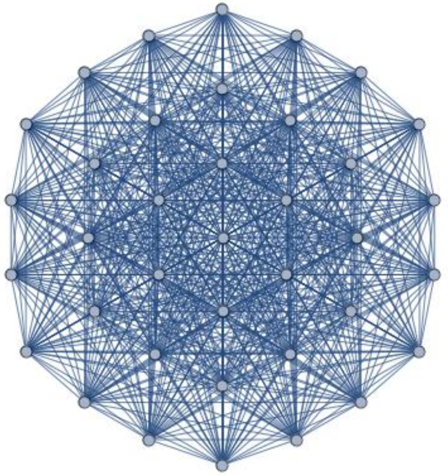
$$\phi^{(n)}_{IPR}(k) = |\langle 0|\phi^n_k\rangle|^4 + |\langle +k, -k|\phi^n_k\rangle|^4,$$



Non-Integrable systems – Long Range LMG model

H,J, Lipkin, N. Meshkov, A.J Glick, *Nuc. Phys* **62** 188 (1965)

Lipkin-Meshkov-Glick model (simplified form)



$$H(t) = \frac{2}{N-1}\sum_{i<j}\sigma_i^z\sigma_j^z + [h\cos\omega t + h_0]\sum_i \sigma_i^x$$

$$\rightarrow H(t) \sim \frac{2J}{N}\left(S^z\right)^2 + h(t)S^x$$

$$\left[S^2, H(t)\right] = 0$$

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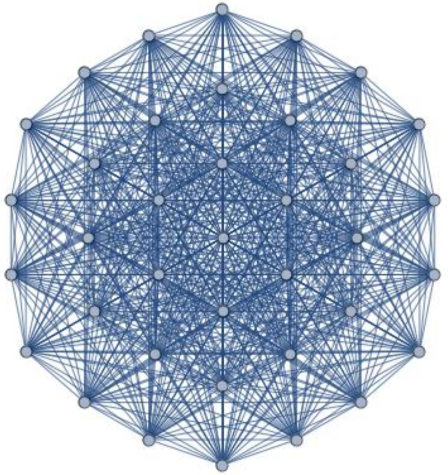

Unitary Transformation:

$$\hat{U}(t) = \exp\left[i\frac{h}{\omega}\sin(\omega t)S^x\right].$$

$$\tilde{H}(t) = -\frac{\hat{S}^2}{N-1} + \frac{\left(\hat{S}^x\right)^2}{N-1} - 2h_0\hat{S}^x - \frac{J_0(\eta)}{N-1}\left[\left(\hat{S}^z\right)^2 - \left(\hat{S}^y\right)^2\right] - \frac{2}{N-1}\left[\left(\hat{S}^z\right)^2 - \left(\hat{S}^y\right)^2\right]\sum_{k=1}^{\infty} J_{2k}(\eta)\cos\left(2k\omega t\right)$$

$$-\frac{2}{N-1}\left\{\hat{S}^y, \hat{S}^z\right\}\sum_{k=1}^{\infty} J_{2k-1}(\eta)\sin\left[(2k-1)\omega t\right].$$

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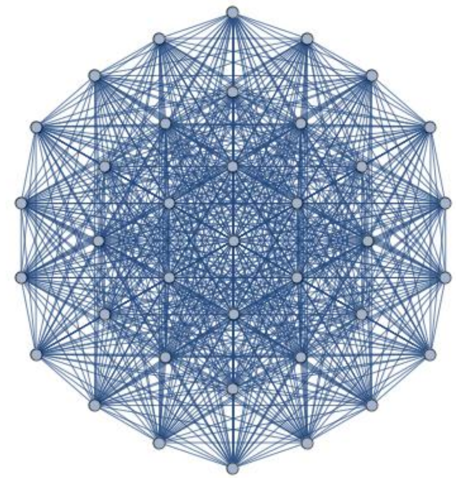

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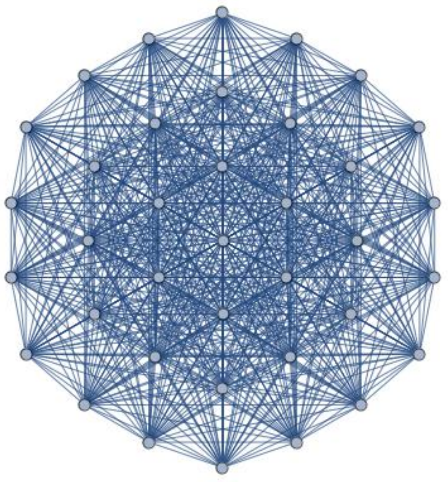

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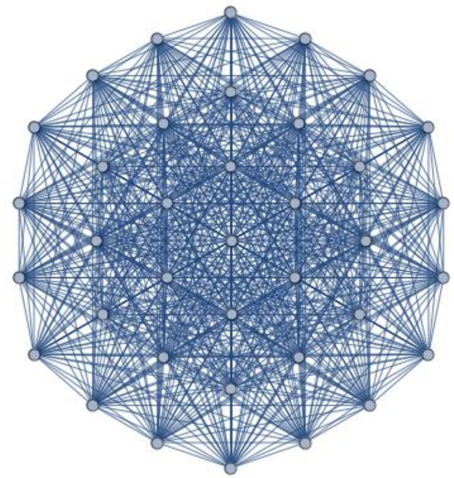

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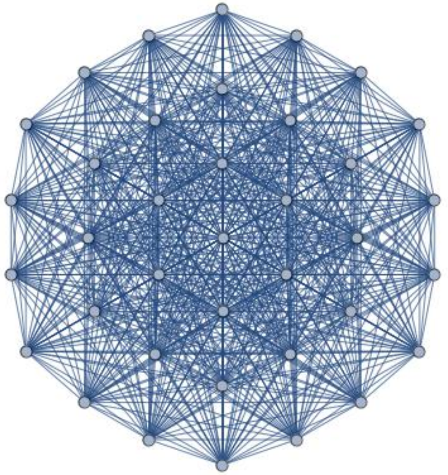

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$$\text{If } S^2 = \frac{N}{2}\left(\frac{N}{2}+1\right) \implies \text{Eigs}\left(\text{S}^{\text{z}}\right) \in \left\{-\frac{\text{N}}{2}, -\frac{\text{N}}{2}+1, \cdots, \frac{\text{N}}{2}-1, \frac{\text{N}}{2}\right\}$$

$$\text{Evecs}\left(S^z\right) \in \left\{\,|-N\rangle\,, |-N+1\rangle\,, \ldots, |0\rangle\,, |1\rangle\,, |2\rangle\,, \ldots, |N-1\rangle\,, |N\rangle\,\right\}$$

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M. Rahaman, T. Mori and A. Roy
arXiv:2308.03622

IPR comparison of Ising model with LMG (N=100)

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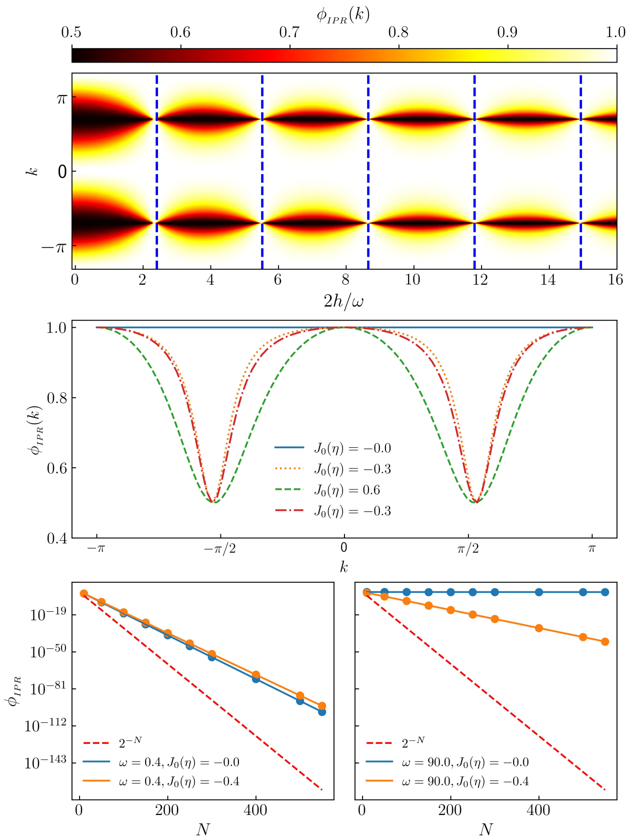

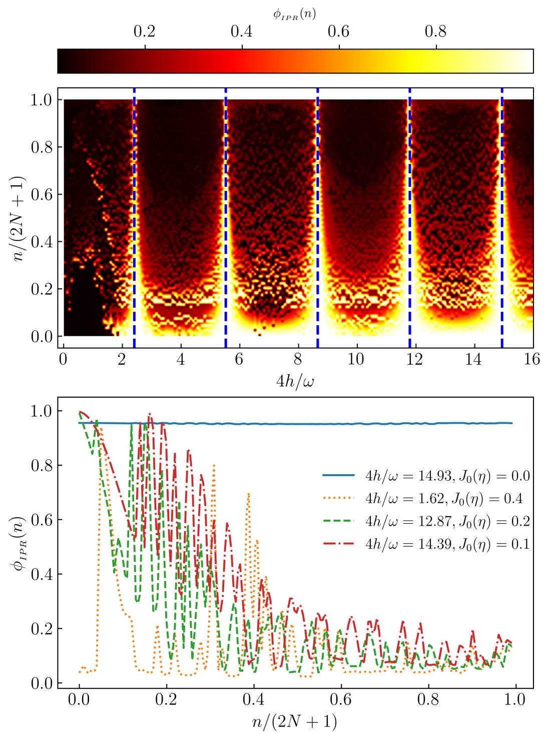

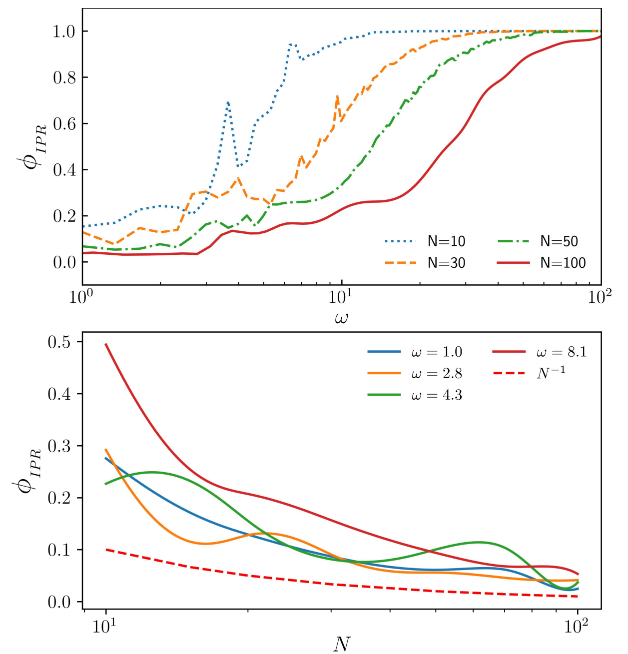

Non-Integrable systems – Long Range LMG model

M. Rahaman, T. Mori and A. Roy
arXiv:2308.03622

IPR comparison of Ising model with LMG (N=100)

$$H(t) = -\frac{1}{2}\left[J\sum_i \sigma_i^x\sigma_{i+1}^x + h(t)\sum_i \sigma_i^z\right]$$

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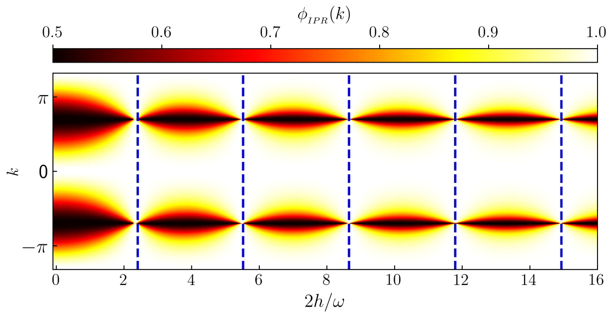

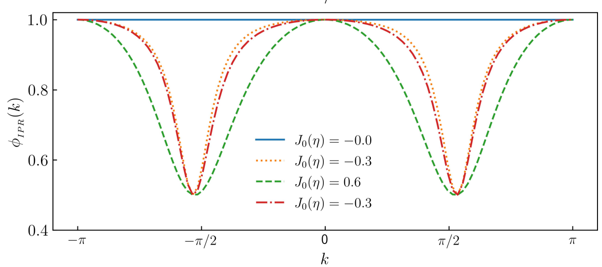

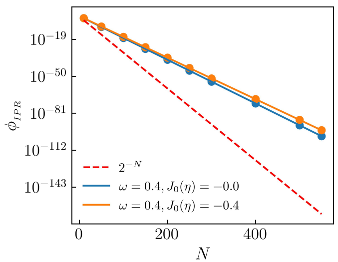

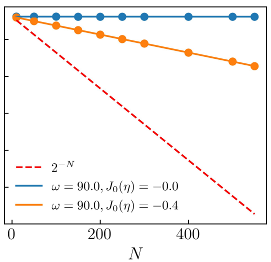

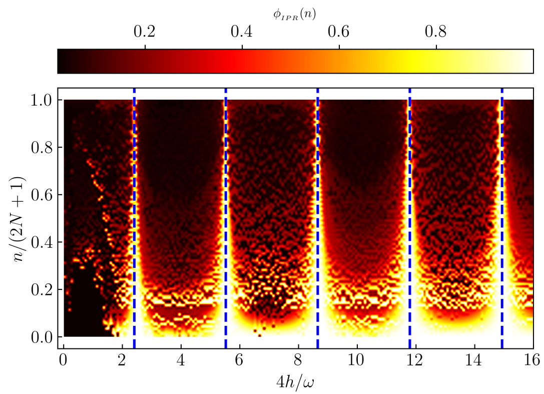

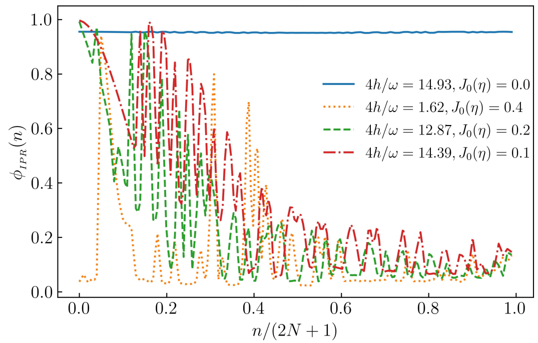

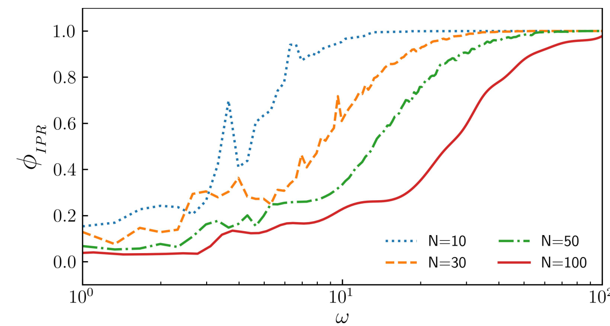

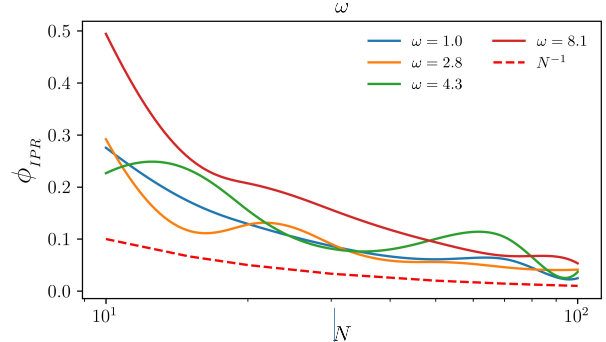

Low freq thermalization, consistent with
A. Russomanno, R. Fazio and G. E. Santoro, EPL 110 (3), 37005 (2015)

Non-Integrable systems – Long Range LMG model

M. Rahaman, T. Mori and A. Roy *arXiv:2308.03622*

G. Engelhardt, V. M. Bastidas, C. Emary, and T. Brandes Phys. Rev. E **87**, 052110 (2013)

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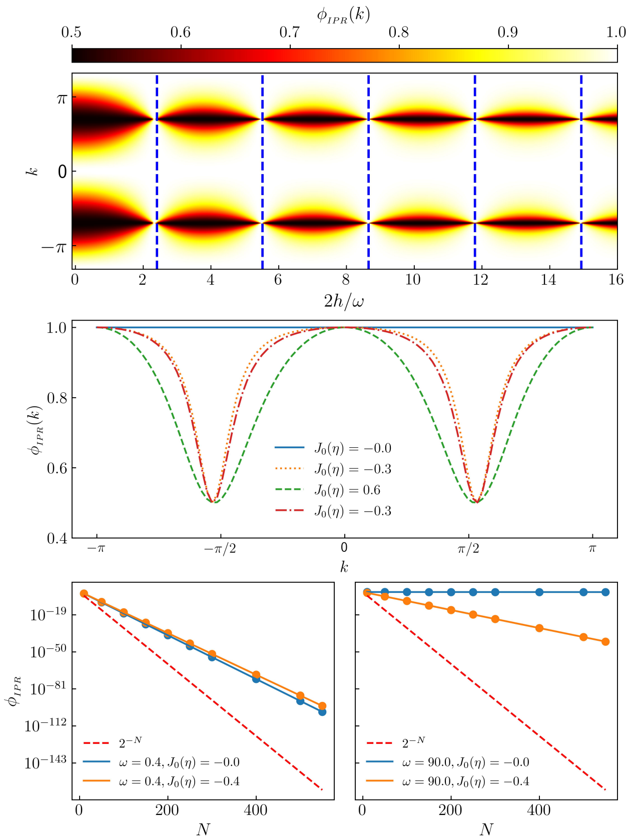

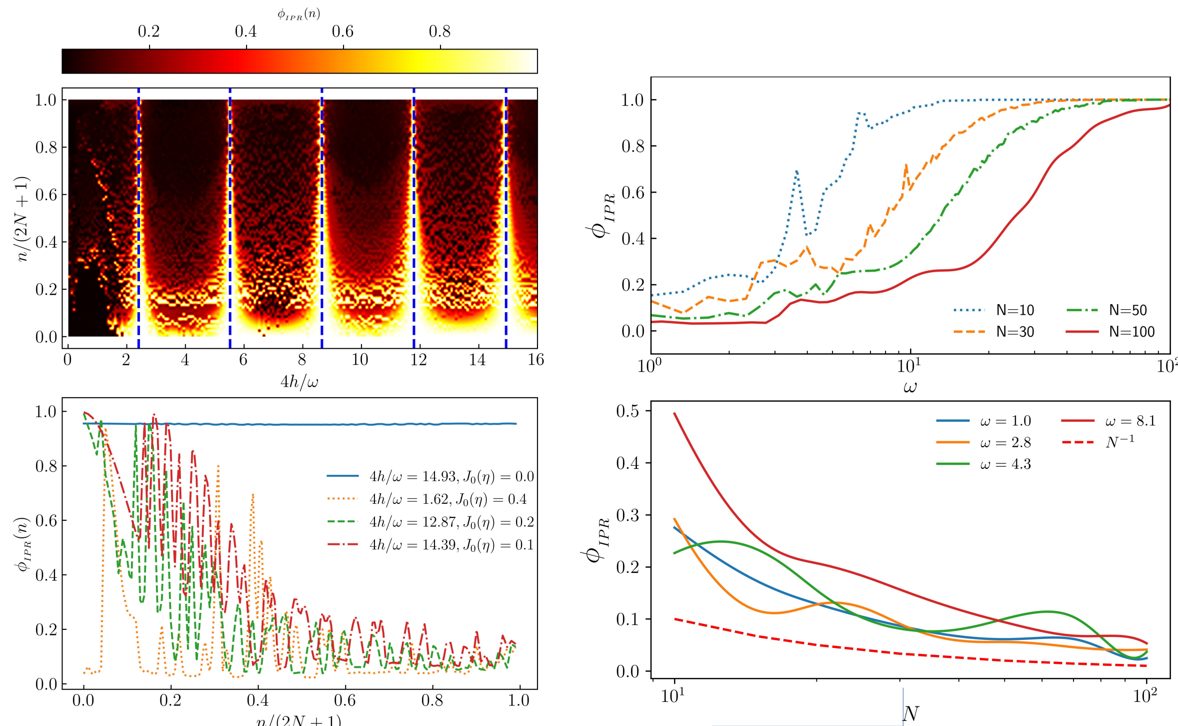

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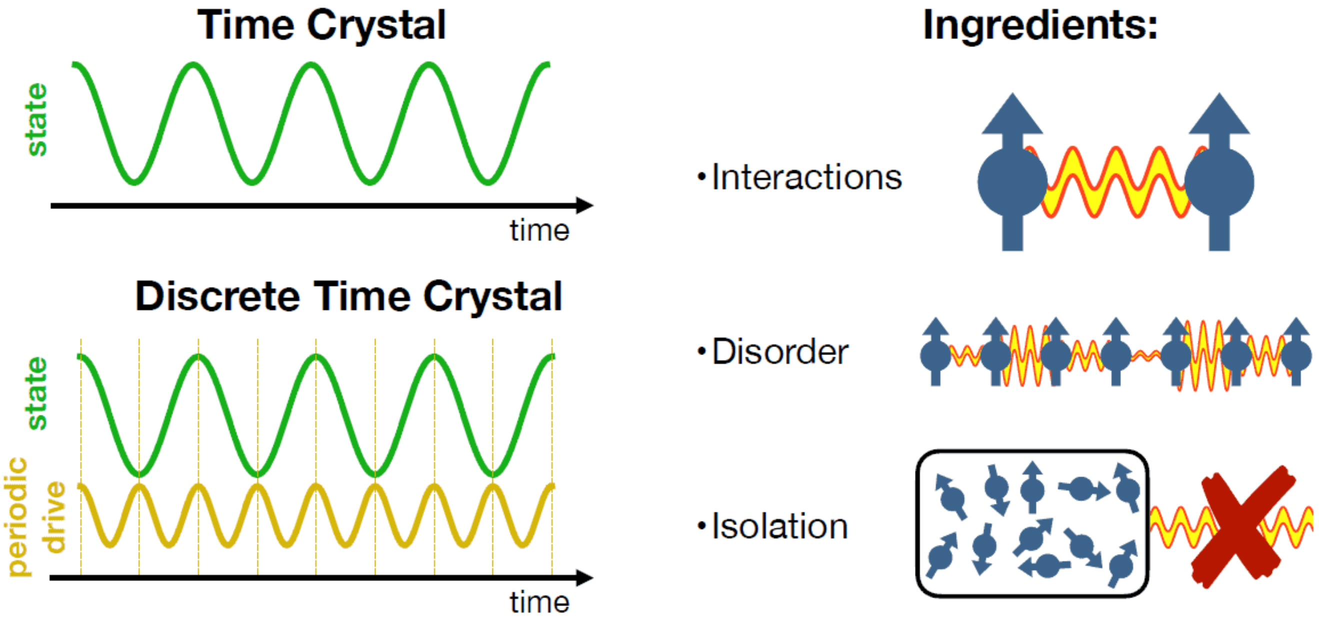
DMBL and Time Crystals

[Wilczek, Frank (2012). PRL. 109 (16): 160401.](#)

[Else, Bauer and Nayak (2016) arXiv:1603.08001](#)

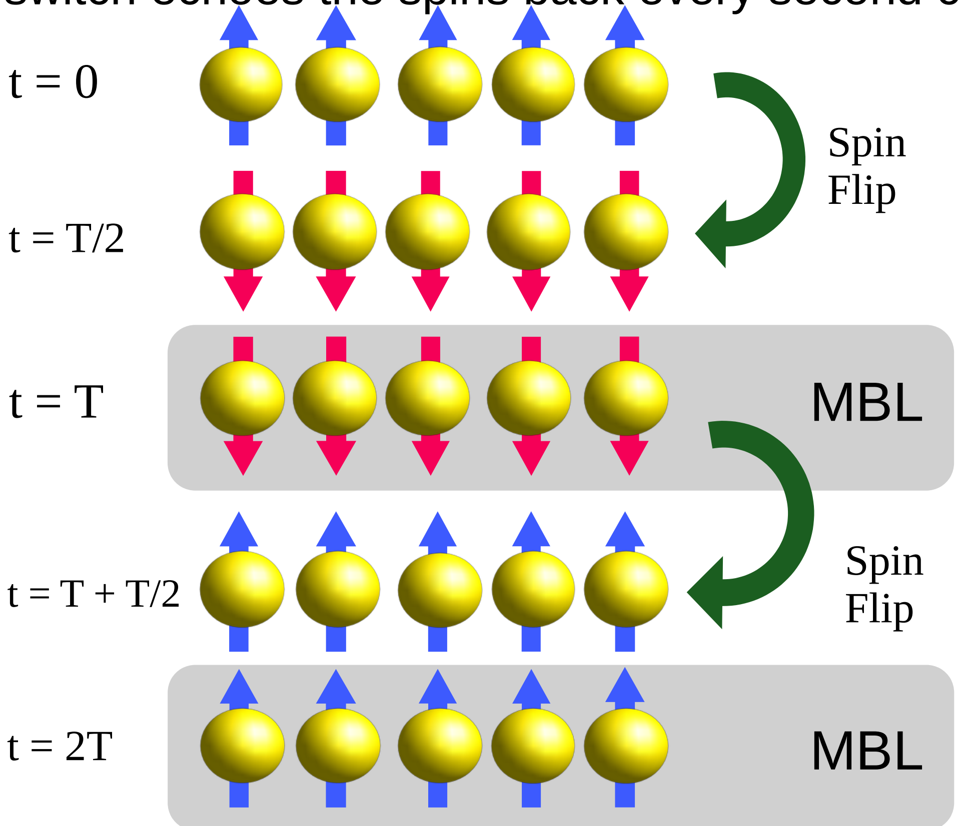
- Discrete time crystals are a novel state of matter where the quasistationary state of a driven system spontaneously breaks the periodicity of the drive.
- Similar to how Z2 symmetry is broken in the Transverse Field Ising Model.



DMBL and Time Crystals

[Rahaman, Sakurai and Roy (2024): arXiv:2309.16523](https://arxiv.org/abs/2309.16523)

- Square wave switches can alternate between two Hamiltonians, one that can flip the spins, and the DMBL Hamiltonian that ensures that they stay flipped until the switch echoes the spins back every second cycle.



$$\hat{H}_1 = \frac{\pi}{T}\sum_i \hat{\sigma}_i^x$$

$$\hat{H}_2(t) = J_0 \sum_{ij} \frac{1}{|i-j|^\beta}\hat{\sigma}_i^y\hat{\sigma}_j^y + h\left(\omega t\right)\sum_i \hat{\sigma}_i^z$$

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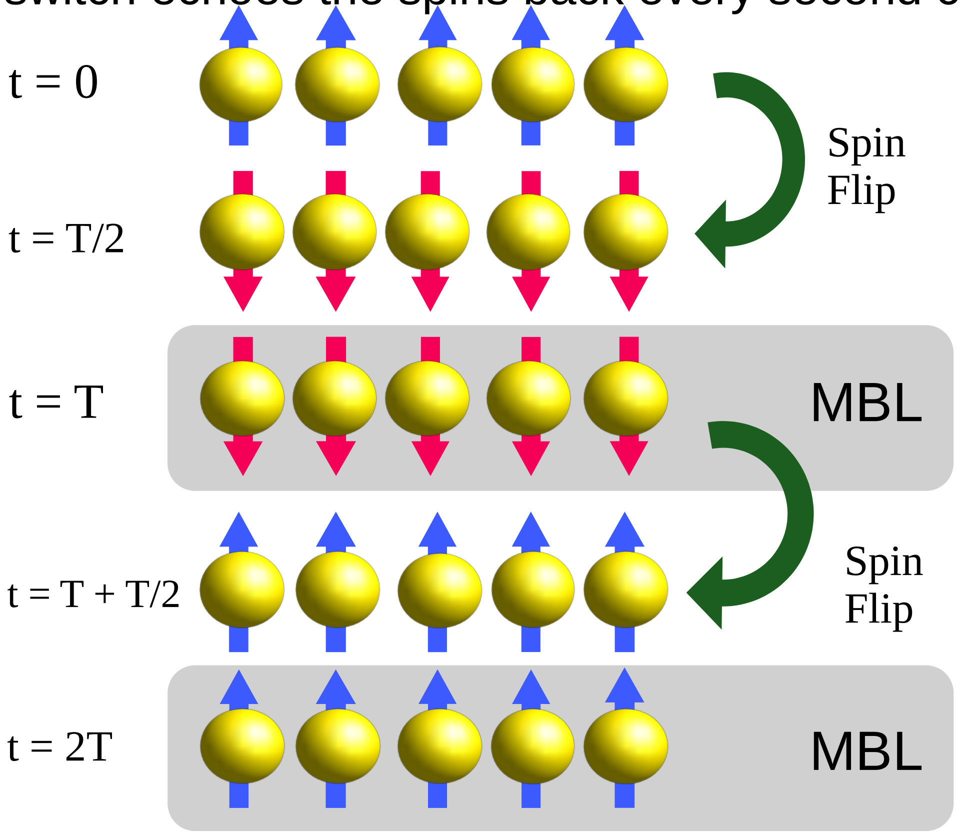

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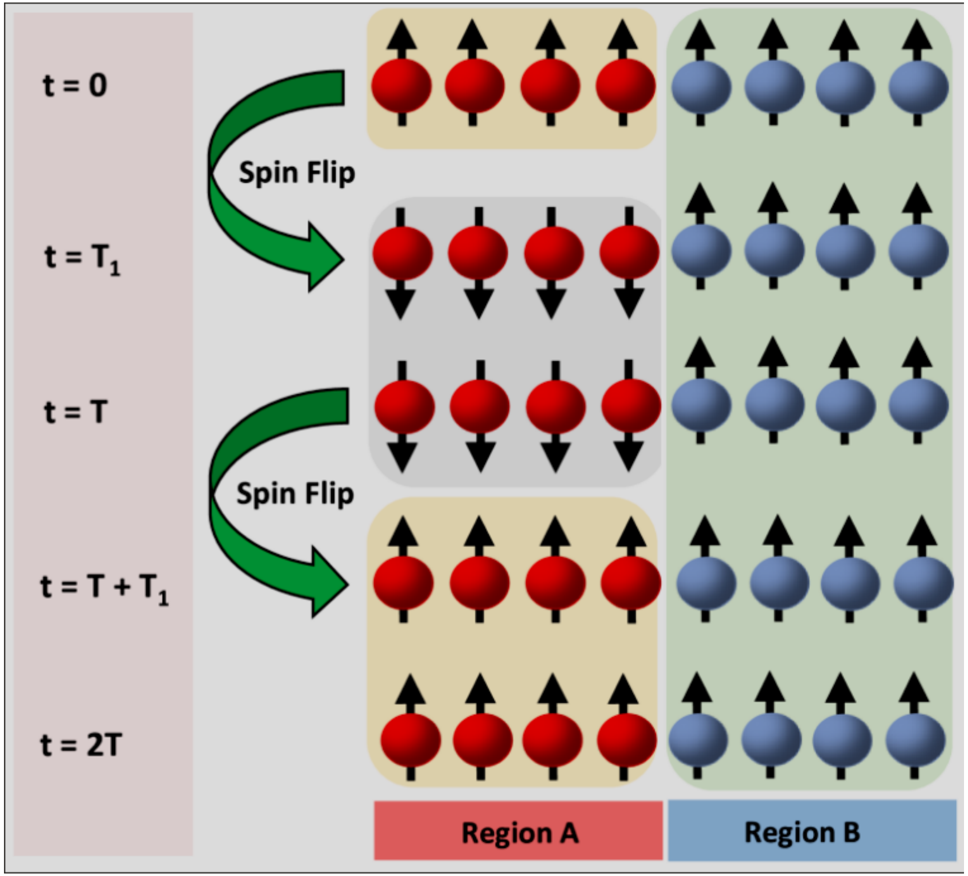
Buyskikh benchmarking:

$\beta = 0$	all-to-all
$0 \leq \beta < 1$	long-range
$1 \leq \beta < 2$	intermediate-range
$2 \leq \beta < \infty$	short-range
$\beta \to \infty$	nearest-neighbor

DMBL and Time Crystals

Rahaman, Sakurai and Roy (2024): arXiv:2309.16523

- The DMBL also ensures rigidity of the time crystal. Spatial variations can produce Chimera-like states where synchronized DTC and asynchronous DMBL co-exist!



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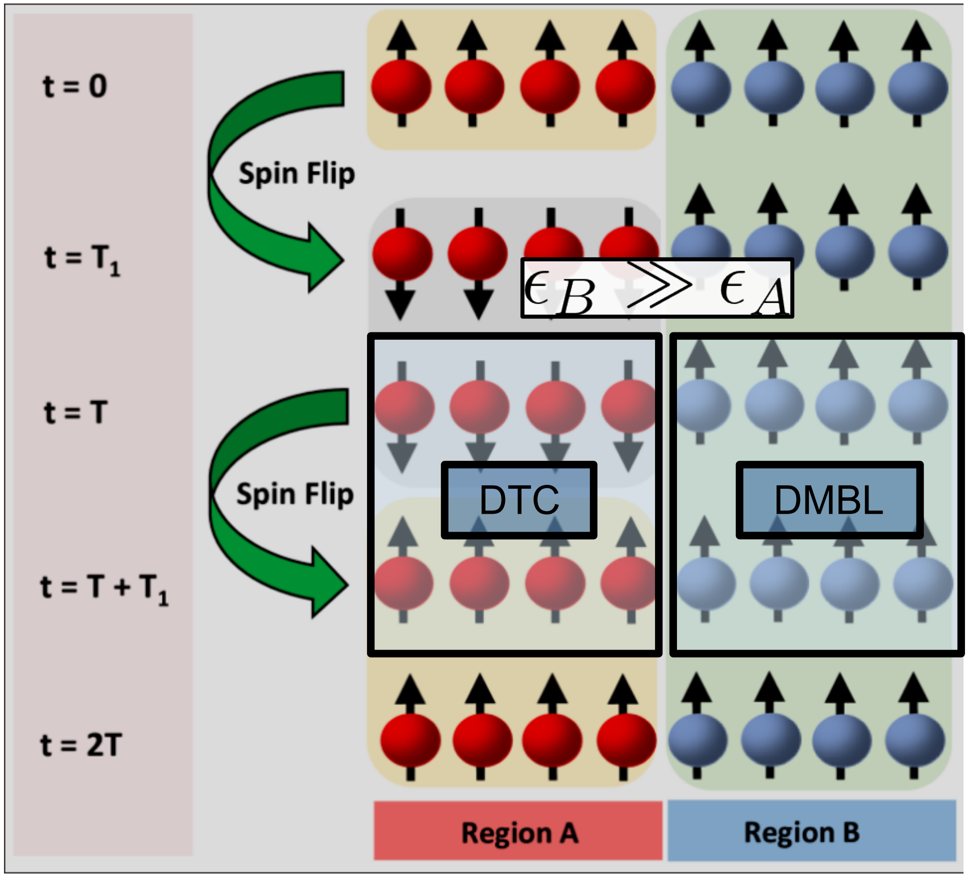

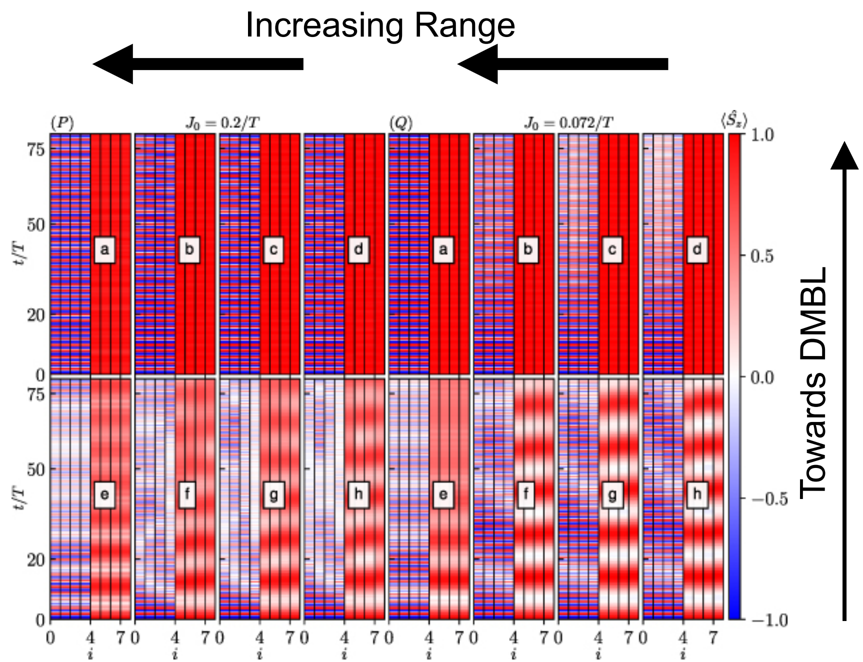

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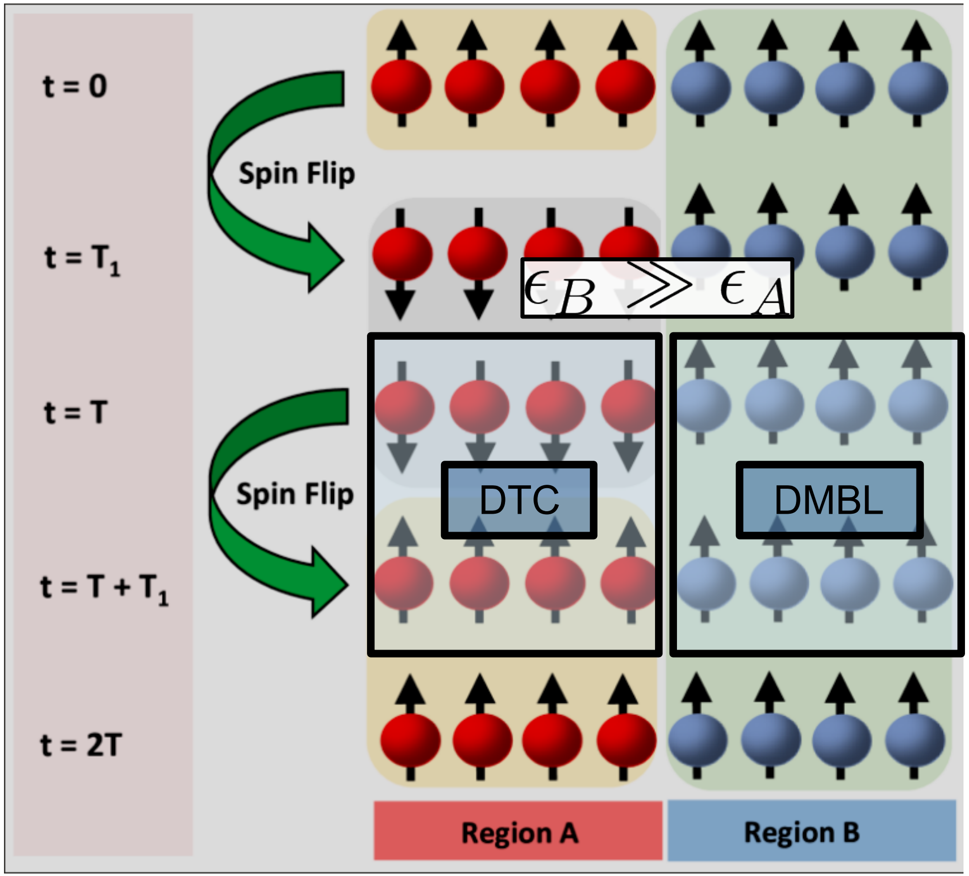

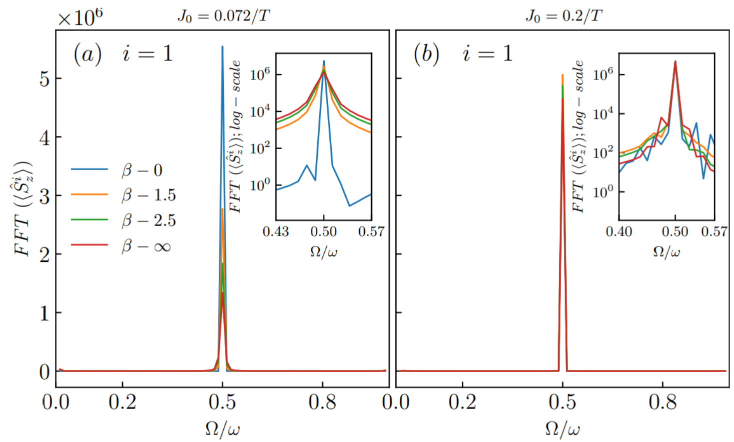

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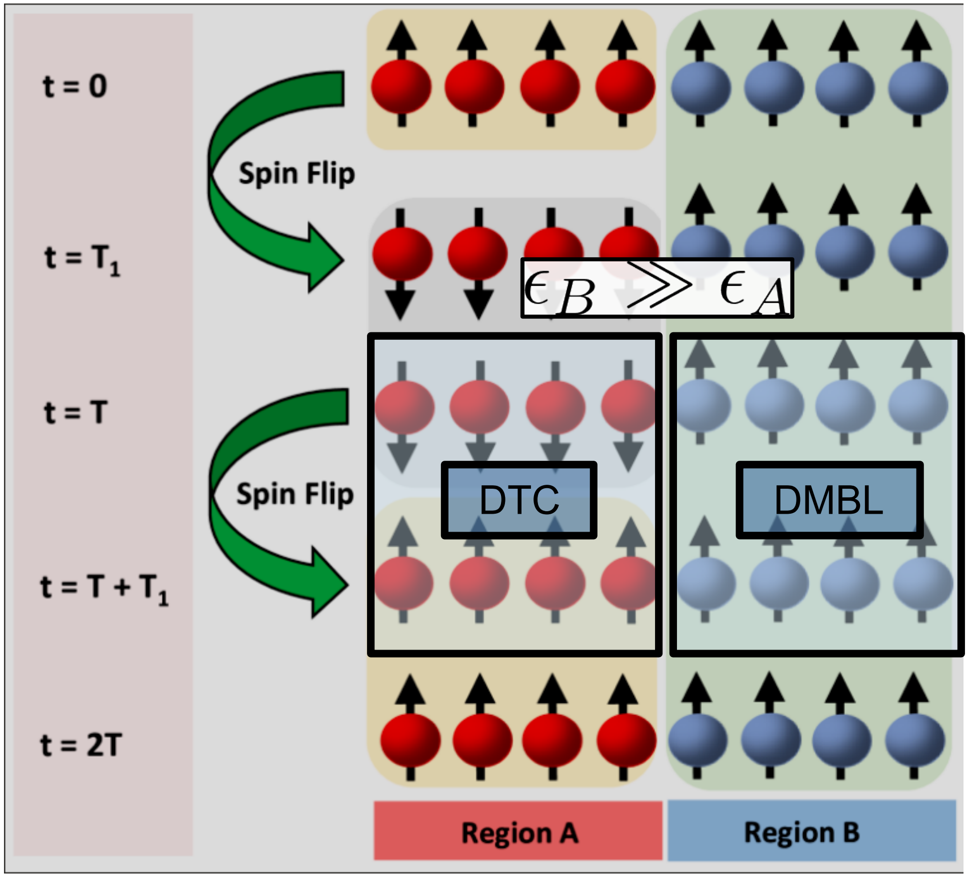

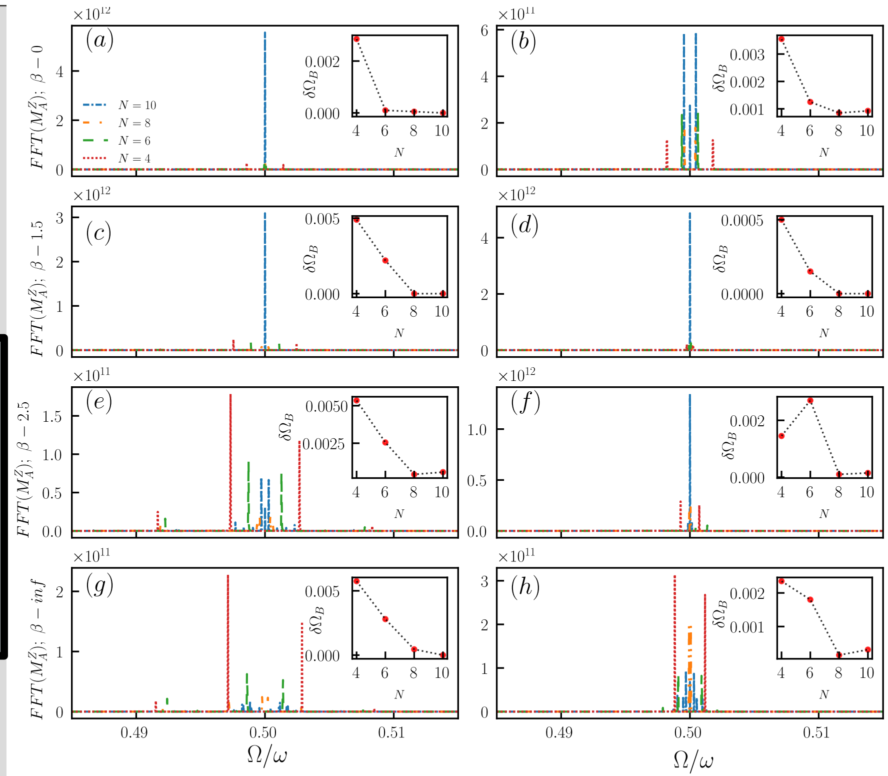

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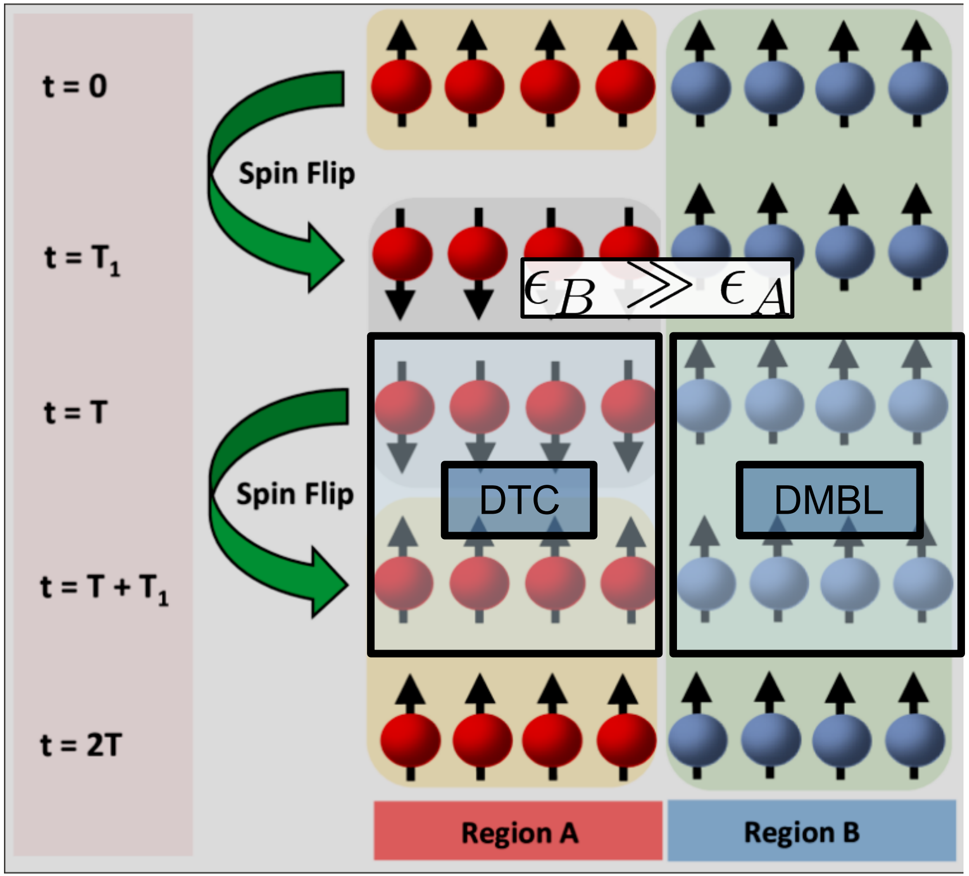

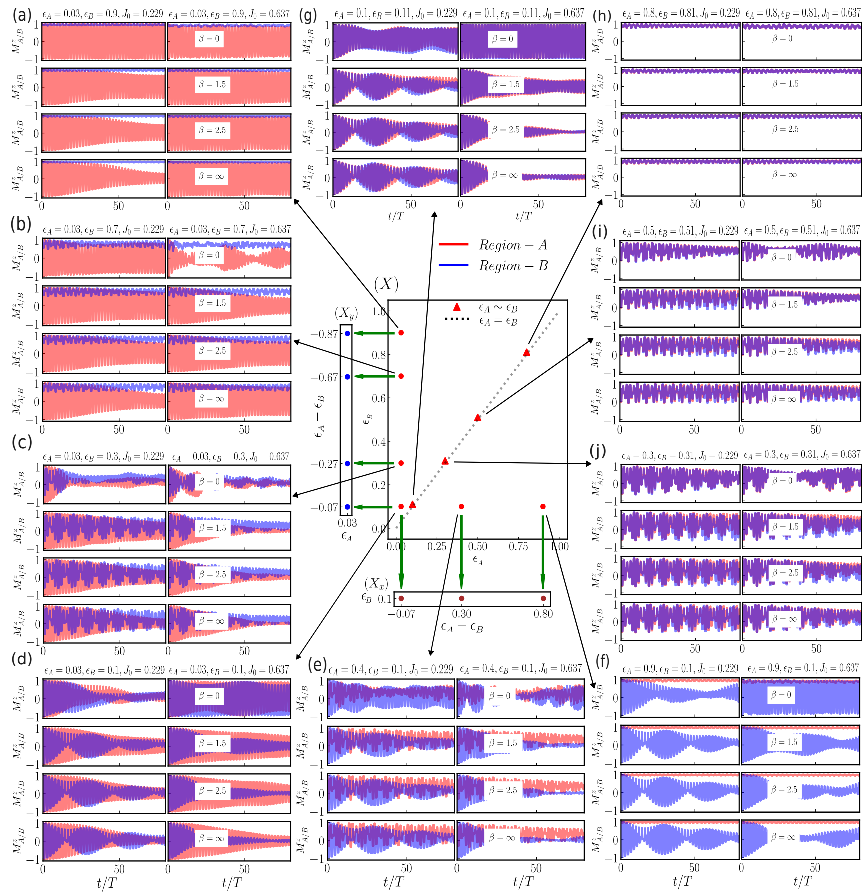

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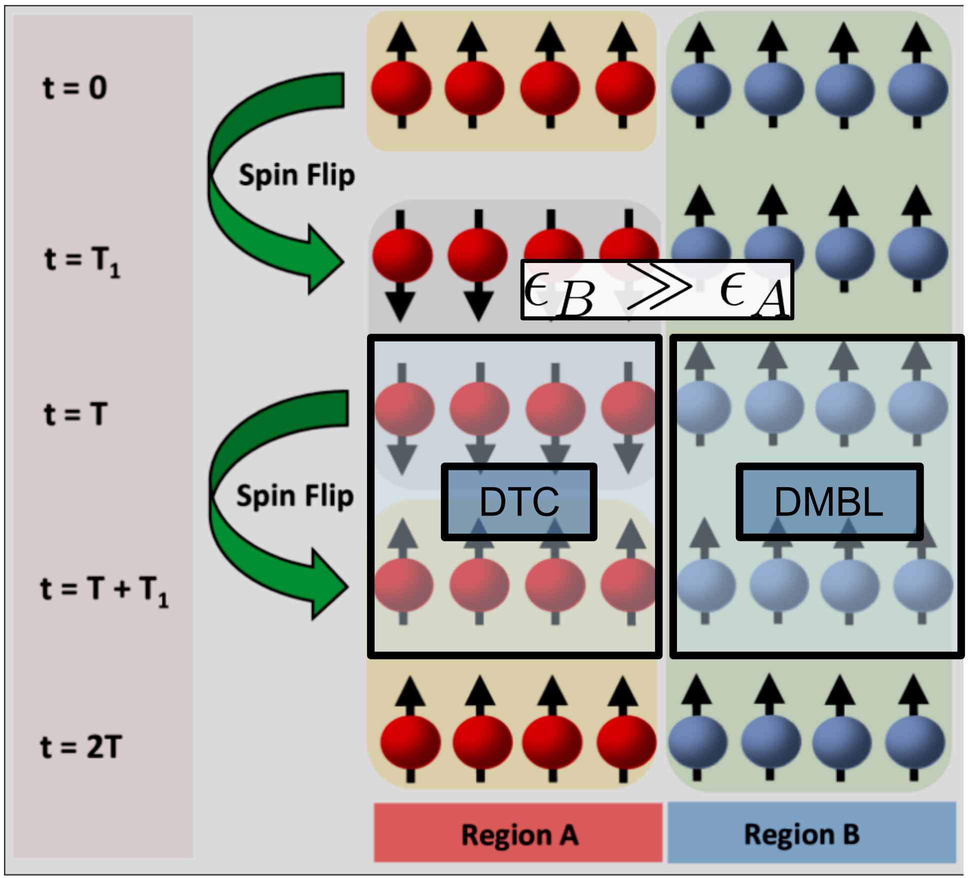

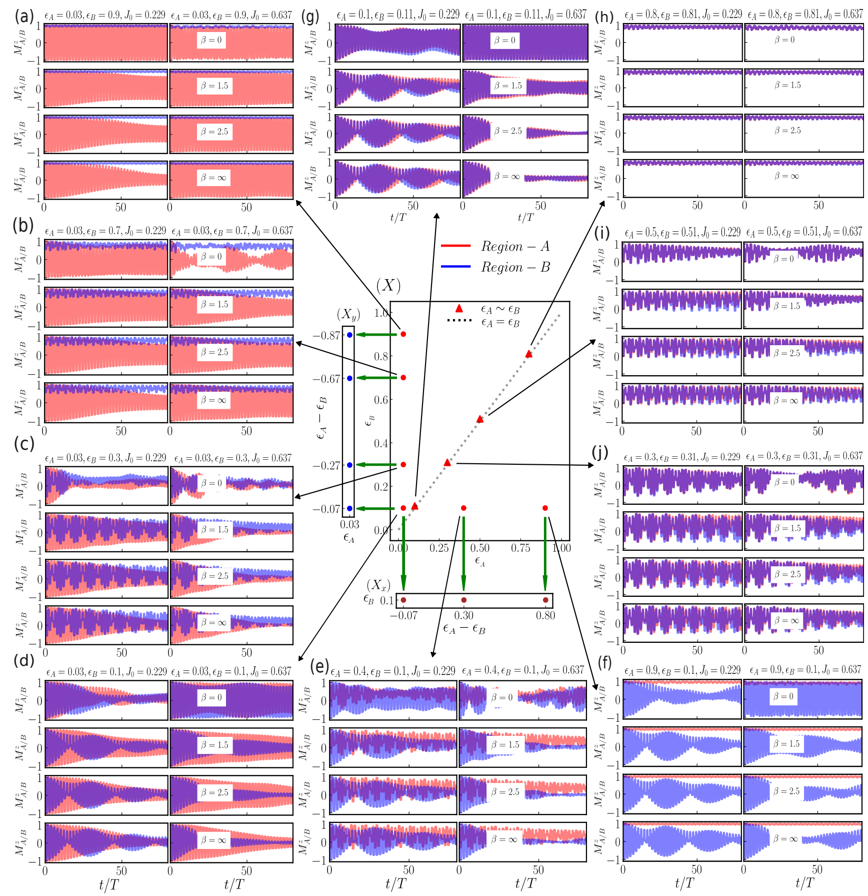

DMBL and Time

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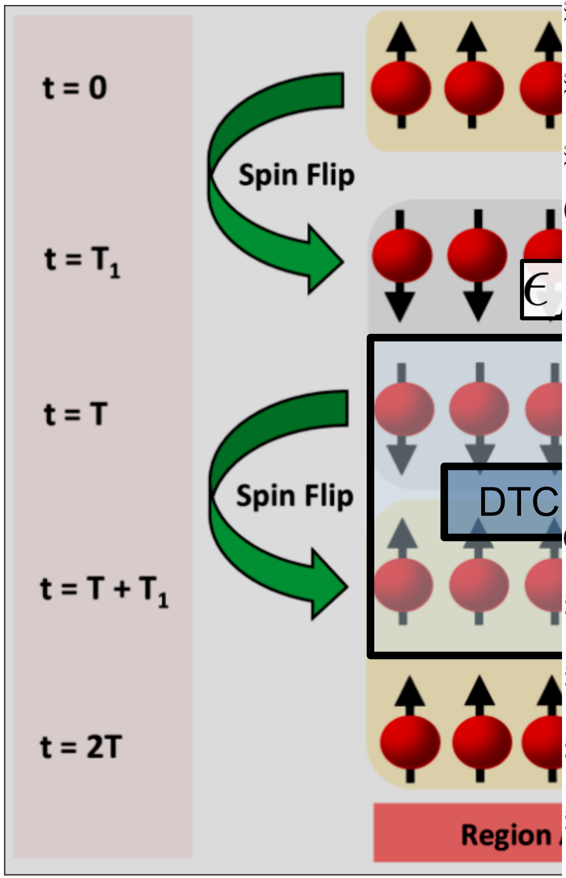
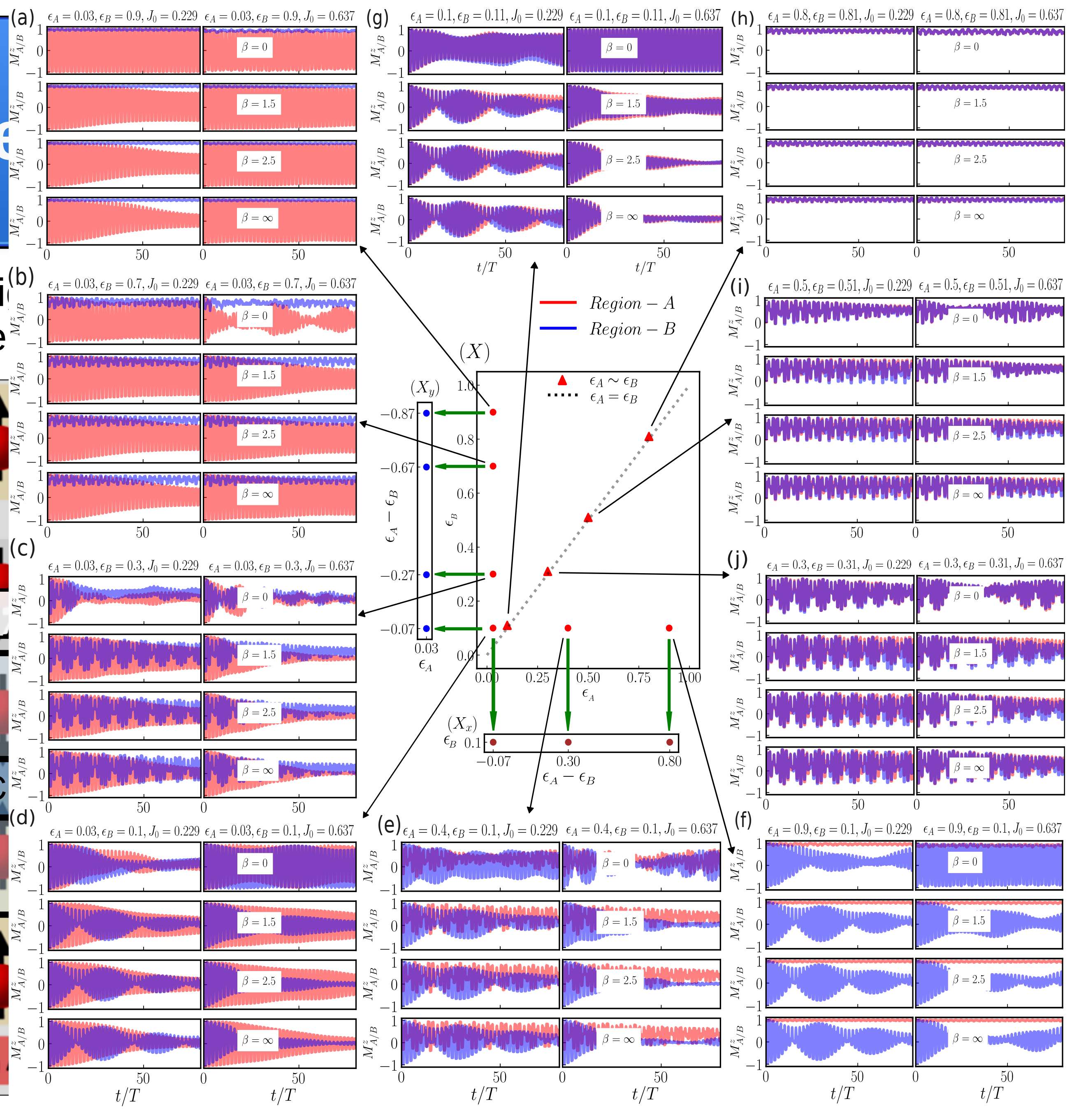




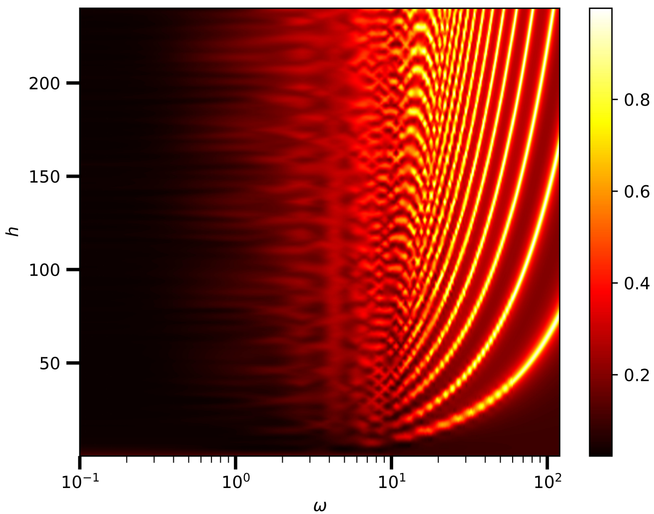
Outlook:

A. Roy and A. Das, *arXiv:1405.3966*

M. Rahaman, T. Mori and A. Roy *arXiv:2308.03622*

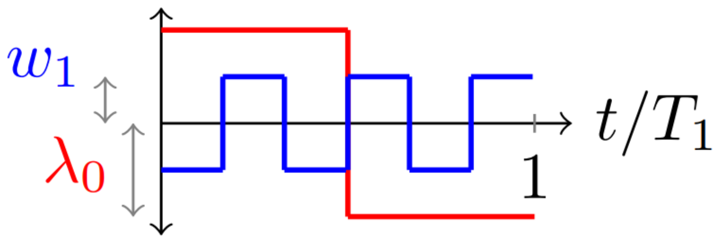
Rahaman, Sakurai and Roy (2024): *arXiv:2309.16523*

1) Can DMBL be used to create more efficient Mesoscale engines and refrigerators?



More efficient MBL engines?
Yunger-Halpern *et. al. arXiv:1707.07008*

2) Other ways to localize dynamics in DTC – Floquet Flat Bands!



The Floquet modes can all be engineered to be degenerate!
No mobility (zero group velocity)
Banerjee, Choudhury, and Sengupta, *arXiv:2404.06536*

Thank You!

A. Roy and A. Das, *arXiv:1405.3966*

M. Rahaman, T. Mori and A. Roy *ArXiv:2308.03622*

Rahaman, Sakurai and Roy (2024): arXiv:2309.16523

Collaborators:

- Profs. Arnab Das (IACS, Kolkata) and Takashi Mori (Keio U. Japan)
- Dr. Akitada Sakurai (Okinawa Inst. Japan)
- Dr. Mahbub Rahaman (BU)

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- Support from the University Grants Commission Grant F.30-425/2018(BSR)