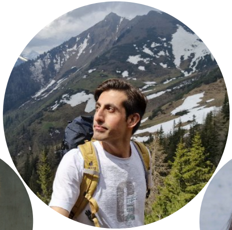
Revisiting Differential-Linear Attacks via a Boomerang Perspective

Applications to AES, Ascon, CLEFIA, SKINNY, PRESENT, KNOT, TWINE, WARP, LBlock, Simeck, and SERPENT

Hosein Hadipour Patrick Derbez Maria Eichlseder

ASK 2024 - Kolkata, India



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Research Gap and Our Contributions

Research Gap

- How to analytically estimate the correlation of DL distinguishers?
- How to (efficiently) find good DL distinguishers?

Contributions

- Generalizing the DLCT framework [Bar+19] for analytical correlation estimation.
- Introducing an efficient method to search for DL distinguishers applicable to:
  - Strongly aligned SPN primitives: AES, SKINNY
  - Weakly aligned SPN primitives: Ascon, SERPENT, KNOT, PRESENT
  - Feistel structures: CLEFIA, TWINE, LBlock, LBlock-s, WARP
  - AndRX designs: Simeck

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Outline

Background

Universal Bound for Data Complexity - I

Theorem (Data Complexity)

Let X_0 and X_1 be two distributions. Given one sample from X_b, the distinguisher $\mathcal{D}$ outputs 1 with probability p if $b = 0$, and outputs 1 with probability q if $b = 1$. Assume that b is chosen uniformly at random from $\{0, 1\}$ and is fixed. Next, we run $\mathcal{D}$ on n samples, and output 1 if the sum of the outcomes is closer to $\mu_0 = np$, and 0 otherwise. If n satisfies the following inequality, then the error probability of the distinguisher is upper bounded by ε:

$$n \geq \max\left(\frac{2(3q + p)\ln\left(\frac{1}{\varepsilon}\right)}{(p - q)^2}, \frac{8p\ln\left(\frac{1}{\varepsilon}\right)}{(p - q)^2}\right).$$

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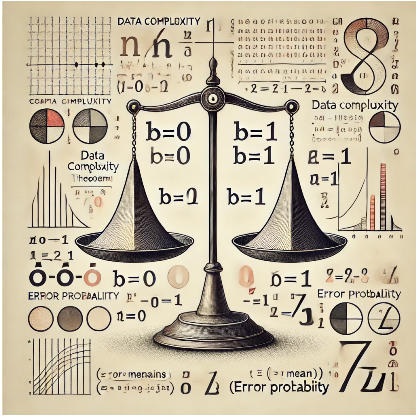
Universal Bound for Data Complexity - II

- $n \geq \max\left(\frac{2(3q+p)\ln\left(\frac{1}{\varepsilon}\right)}{(p-q)^2}, \frac{8p\ln\left(\frac{1}{\varepsilon}\right)}{(p-q)^2}\right).$
- If $p \gg q$, then $p - q \approx p$ then $n \geq \frac{8\ln\left(\frac{1}{\varepsilon}\right)}{p}$.
- If $p = \frac{1}{2} + \frac{c}{2}, q = \frac{1}{2} + \frac{c'}{2}, c \gg c'$, and $c, c' \ll \frac{1}{2}$ then $n \geq \frac{8\ln\left(\frac{1}{\varepsilon}\right)}{c^2}$.



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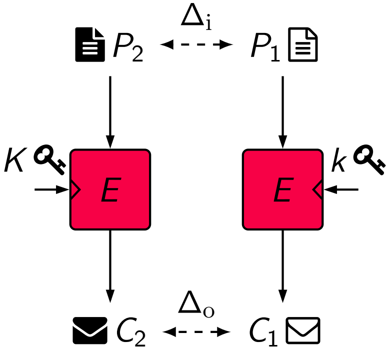
Differential Attacks [BS90]

Input: $E_K, (\Delta_\mathrm{i}, \Delta_\mathrm{o}), N, p = \mathbb{P}(\Delta_\mathrm{i}, \Delta_\mathrm{o})$

Output: 0: **real** cipher, 1: **ideal** cipher

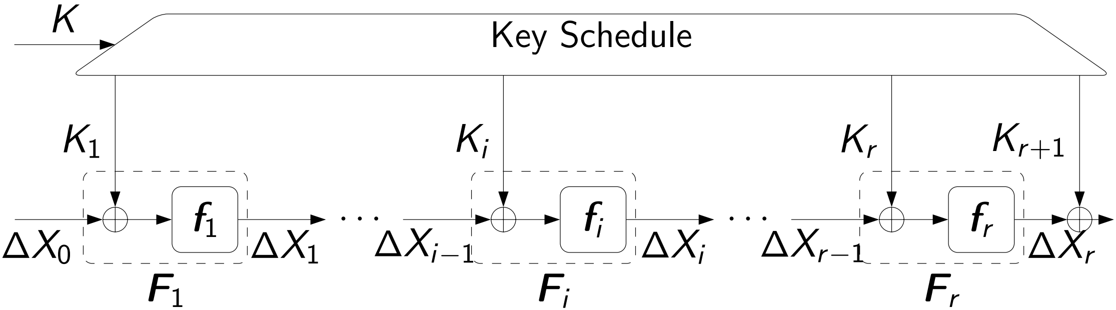
```
Initialize counter T with zero;
for i = 0, ..., N − 1 do
    P_1 ←$ F_2^n;
    C_1 ← E_K(P_1);
    P_2 ← P_1 ⊕ Δ_i;
    C_2 ← E_K(P_2);
    if C_1 ⊕ C_2 = Δ_o then
        T ← T + 1;
if T ~ N(μ = Np, σ^2 = Np(1 − p)) then
    return 0;                     // real cipher
else
    return 1;                     // ideal cipher
```

$N \approx \mathcal{O}(p^{-1})$.



$p = \mathbb{P}(\Delta_\mathrm{i} \rightarrow \Delta_\mathrm{o})$

Analytical Estimation of Differential Probability



$$\mathbb{P}(\Delta X_r = \Delta_r \mid \Delta X_0 = \Delta_0) = \sum_{\Delta_1, \cdots, \Delta_{r-1}} \prod_{i=1}^{r} \mathbb{P}(f_i(X) \oplus f_i(X \oplus \Delta_{i-1}) = \Delta_i).$$

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Difference Distribution Table (DDT) – I

We need a tool to handle the nonlinear operations

Differential Distribution Table (DDT)

For a vectorial Boolean function $S : \mathbb{F}_2^n \to \mathbb{F}_2^m$, the DDT is a $2^n \times 2^m$ table whose rows correspond to the input difference Δ_i to S and whose columns correspond to the output difference Δ_o of S. The entry at index $(\Delta_\mathrm{i}, \Delta_\mathrm{o})$ is

$$\mathtt{DDT}(\Delta_\mathrm{i}, \Delta_\mathrm{o}) = |\{x \in \mathbb{F}_2^n : \ S(x) \oplus S(x \oplus \Delta_\mathrm{i}) = \Delta_\mathrm{o}\}|.$$

$$\mathbb{P}\left(\Delta_\mathrm{i}, \Delta_\mathrm{o}\right) = 2^{-n} \cdot \mathtt{DDT}\left(\Delta_\mathrm{i}, , \Delta_\mathrm{o}\right)$$

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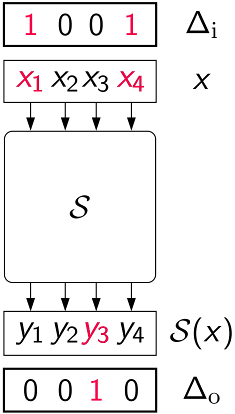
Difference Distribution Table (DDT) – II



$$\mathbb{P}(9, 2) = \frac{4}{16}$$

$\Delta_{\mathrm{i}} \setminus \Delta_{\mathrm{o}}$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	2	2	2	2	0	0	0	0	2	2	2	2
2	0	2	0	2	0	0	0	4	0	2	2	0	0	0	2	2
3	0	2	0	2	0	0	4	0	0	2	2	0	0	0	2	2
4	0	0	0	0	0	0	0	0	0	0	4	4	2	2	2	2
5	0	0	0	0	2	2	2	2	0	0	4	4	0	0	0	0
6	0	2	0	2	0	4	0	0	0	2	2	0	2	2	0	0
7	0	2	0	2	4	0	0	0	0	2	2	0	2	2	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	4	4	4	4
9	0	4	4	0	0	0	0	0	0	4	0	4	0	0	0	0
a	0	0	2	2	2	0	0	2	4	0	0	0	0	2	0	2
b	0	0	2	2	0	2	2	0	4	0	0	0	2	0	2	0
c	0	4	4	0	2	2	2	2	0	0	0	0	0	0	0	0
d	0	0	0	0	2	2	2	2	0	4	0	4	0	0	0	0
e	0	0	2	2	0	2	2	0	4	0	0	0	0	2	0	2
f	0	0	2	2	2	0	0	2	4	0	0	0	2	0	2	0

Linear Attacks [Mat93]

Input: E_K, Given N distinct plaintext-ciphertext pairs (P_i, C_i), $\boldsymbol{c} = \mathbb{C}(\lambda_\mathrm{i}, \lambda_\mathrm{o})$

Output: 0: **real** cipher, 1: **ideal** cipher

Initialize a counter list $V[z] \leftarrow 0$ for $z \in \{0, 1\}$;

for $t = 0, \dots, N - 1$ **do**

  $b_1 \leftarrow \lambda_\mathrm{i} \cdot P_t$;

  $b_2 \leftarrow \lambda_\mathrm{o} \cdot C_t$;

  $V[b_1 \oplus b_2] \leftarrow V[b_1 \oplus b_2] + 1$;

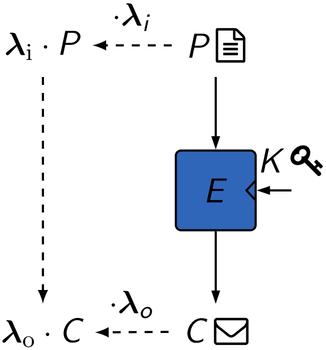
if $V[0] \sim \mathcal{N}(\mu_0 = N\frac{1+\boldsymbol{c}}{2}, \sigma_0^2 = \frac{N(1-c^2)}{4})$. **then**

  return 0; `// real cipher`

else

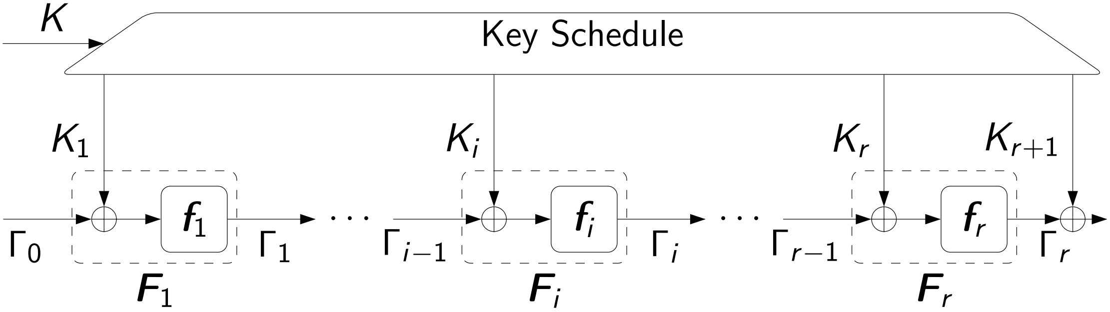
  return 1; `// ideal cipher`

$$N = \mathcal{O}(\boldsymbol{c}^{-2}).$$



$$\boldsymbol{c} = 2 \cdot \mathbb{P}(\boldsymbol{\lambda}_\mathrm{i} \cdot P \oplus \boldsymbol{\lambda}_\mathrm{o} \cdot C = 0) - 1$$

Analytical Estimation of Correlation



$$\mathbb{C}(\Gamma_0, \Gamma_{r+1}) \approx (-1)^{(\Gamma_0 \cdot K_1 \oplus \cdots \oplus \Gamma_r \cdot K_{r+1})} \prod_{i=1}^{r} \mathbb{C}_{\boldsymbol{f}_i}(\Gamma_{i-1}, \Gamma_i).$$

Linear Approximation Table ($\texttt{LAT}$) – I

We need a metric to measure the quality of a linear approximation.

Linear Approximation Table ($\texttt{LAT}$)

For a vectorial Boolean function $S : \mathbb{F}_2^n \to \mathbb{F}_2^m$, the $\texttt{LAT}$ of S is a $2^n \times 2^m$ table whose rows correspond to the input mask λ_i to S and whose columns correspond to the output mask λ_o of S. The entry at index $(\lambda_\mathrm{i}, \lambda_\mathrm{o})$ is

$$\texttt{LAT}(\lambda_\mathrm{i}, \lambda_\mathrm{o}) = |\texttt{LAT}_0(\lambda_\mathrm{i}, \lambda_\mathrm{o})| - |\texttt{LAT}_1(\lambda_\mathrm{i}, \lambda_\mathrm{o})|,$$

where $\texttt{LAT}_b(\lambda_\mathrm{i}, \lambda_\mathrm{o}) = \{x \in \mathbb{F}_2^n : \ \lambda_\mathrm{i} \cdot x \oplus \lambda_\mathrm{o} \cdot S(x) = b\}$.

$\mathbb{C}\left(\lambda_\mathrm{i}, \lambda_\mathrm{o}\right) = 2^{-n} \cdot \texttt{LAT}\left(\lambda_\mathrm{i}, \lambda_\mathrm{o}\right)$

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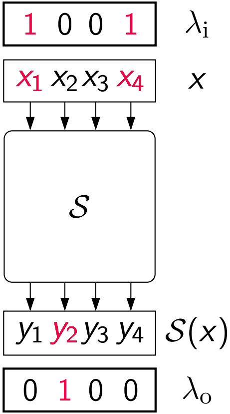
Linear Approximation Table (LAT) – II



$$\mathbb{C}(9,4) = \frac{8}{16}$$

$\lambda_i \setminus \lambda_o$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	4	-4	0	-8	-4	-4	0	0	4	-4	-8	0	4	4
2	0	0	0	0	0	0	0	0	0	8	8	0	0	8	-8	0
3	0	-8	4	4	0	0	-4	4	0	0	-4	4	-8	0	-4	-4
4	0	4	0	4	0	4	8	-4	0	4	0	4	-8	-4	0	4
5	0	4	-4	-8	0	-4	-4	0	0	4	-4	8	0	-4	-4	0
6	0	-4	8	4	0	-4	0	-4	0	4	0	4	8	-4	0	4
7	0	4	4	0	0	-4	4	-8	0	-4	-4	0	0	4	-4	-8
8	0	0	0	0	0	0	0	0	0	0	8	8	0	0	8	-8
9	0	0	-4	4	**8**	0	-4	-4	0	0	4	-4	0	-8	-4	-4
a	0	8	0	8	0	-8	0	8	0	0	0	0	0	0	0	0
b	0	0	-4	4	-8	0	-4	-4	0	8	-4	-4	0	0	4	-4
c	0	4	0	4	0	4	-8	-4	8	-4	0	4	0	4	0	4
d	0	4	4	0	-8	4	-4	0	-8	-4	4	0	0	-4	-4	0
e	0	4	8	-4	0	4	0	4	8	4	0	-4	0	-4	0	-4
f	0	-4	-4	0	-8	-4	4	0	8	-4	4	0	0	-4	-4	0

Boomerang Distinguishers [Wag99]

Input: $E_K, (\Delta, \nabla), N, P = \mathbb{P}(P_3 \oplus P_4 = \Delta)$

Output: 0: **real** cipher, 1: **ideal** cipher

Initialize counter T with zero;

for $i = 0, \ldots, N-1$ **do**

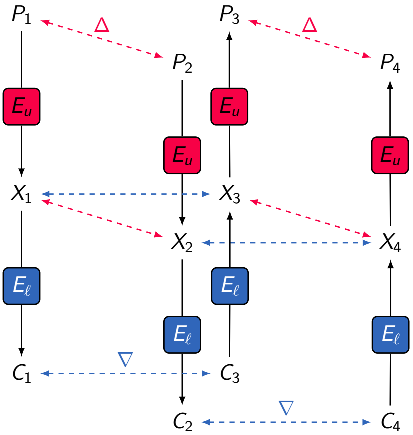
- $P_1 \xleftarrow{\$} \mathbb{F}_2^n$; $P_2 = P_1 \oplus \Delta$;
- $C_1 \leftarrow E_K(P_1)$, $C_2 \leftarrow E_K(P_2)$;
- $C_3 \leftarrow C_1 \oplus \nabla$, $C_4 \leftarrow C_2 \oplus \nabla$;
- $P_3 \leftarrow D_K(C_3)$, $P_4 \leftarrow D_K(C_4)$;
- **if** $P_3 \oplus P_4 = \Delta$ **then**
  - $T \leftarrow T + 1$;

if $T \sim \mathcal{N}(\mu = NP, \sigma^2 = NP(1-P))$ **then**

- **return** 0; `// real cipher`

else

- **return** 1; `// ideal cipher`



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Probability of Boomerang Distinguishers [Wag99]

$$\Delta \longrightarrow \boxed{E : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^n} \longrightarrow \nabla$$

$$0 \lneq \mathbb{P}(\Delta \xrightarrow{E} \nabla) \lll 2^{-n}$$

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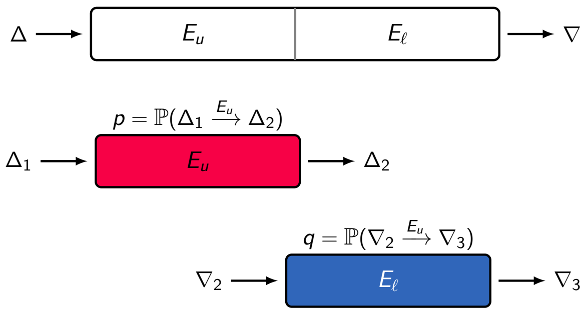
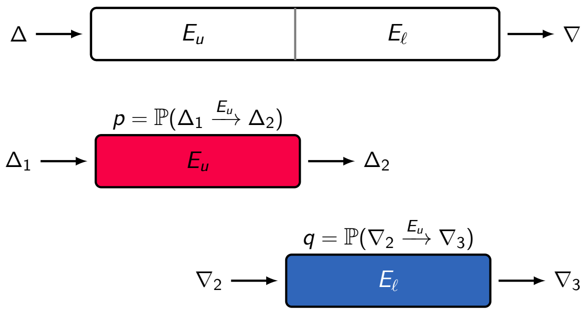
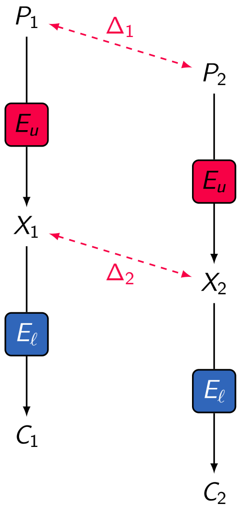
Probability of Boomerang Distinguishers [Wag99]



Probability of Boomerang Distinguishers [Wag99]





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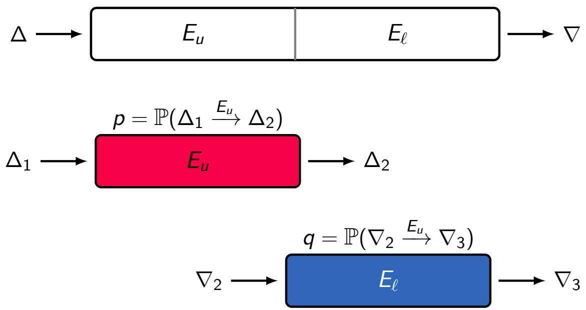
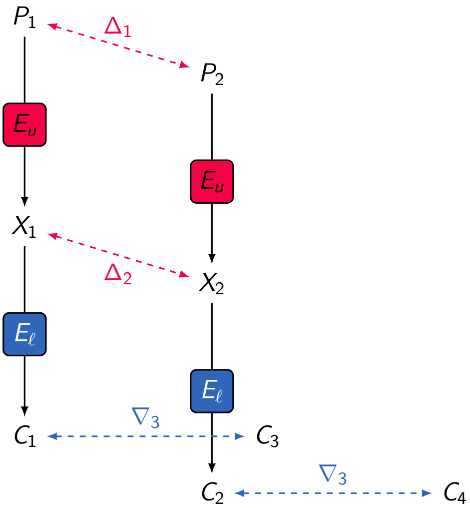
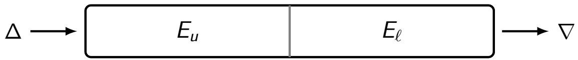


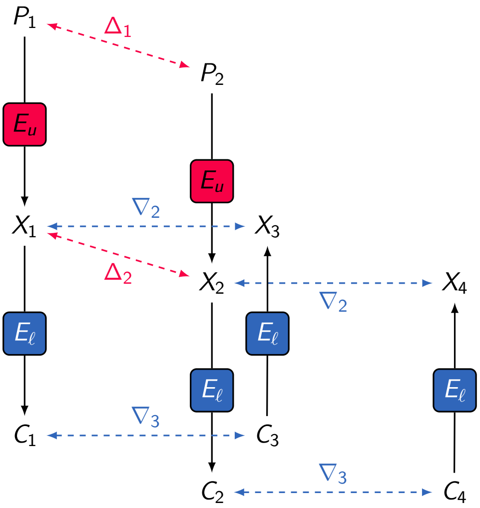


Probability of Boomerang Distinguishers [Wag99]

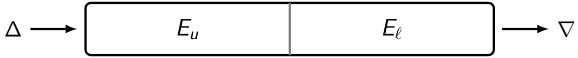


$p = \mathbb{P}(\Delta_1 \xrightarrow{E_u} \Delta_2)$

$q = \mathbb{P}(\nabla_2 \xrightarrow{E_u} \nabla_3)$



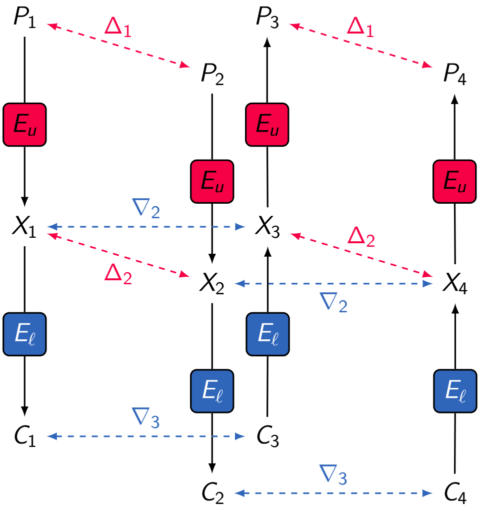
Probability of Boomerang Distinguishers [Wag99]



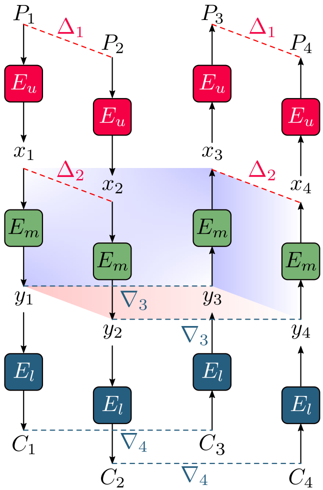
$p = \mathbb{P}(\Delta_1 \xrightarrow{E_u} \Delta_2)$

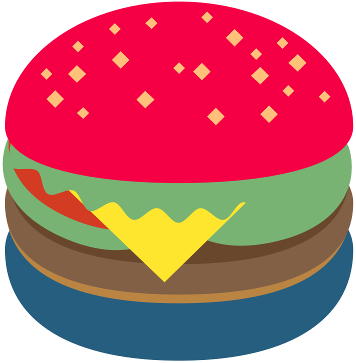
$q = \mathbb{P}(\nabla_2 \xrightarrow{E_u} \nabla_3)$

$\boxed{\mathbb{P}(P_3 \oplus P_4 = \Delta_1) = p^2 q^2}$



Sandwiching the Differentials! [DKS10; DKS14]





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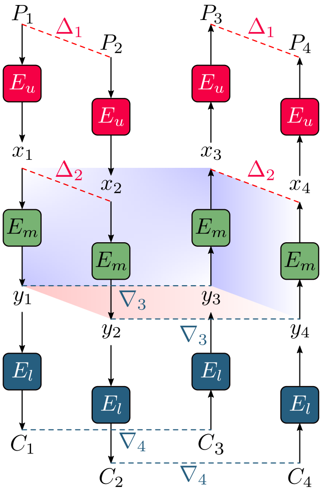
Sandwiching the Differentials! [DKS10; DKS14]



$$\mathbb{P}(P_3 \oplus P_4 = \Delta_1) \approx p^2 \times r \times q^2$$

$$r = \mathbb{P}(\Delta_2 \rightleftarrows \nabla_3)$$

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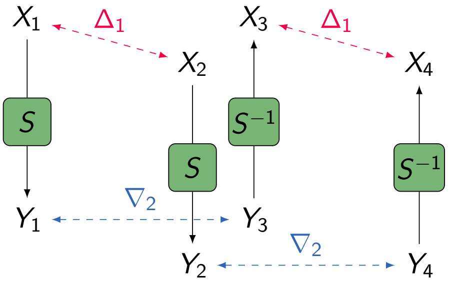
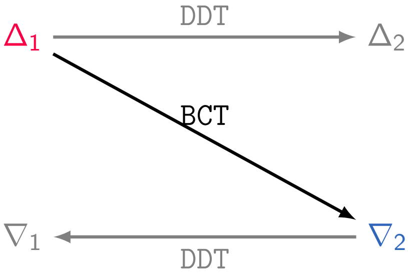
Boomerang Connectivity Table (BCT) [Cid+18]

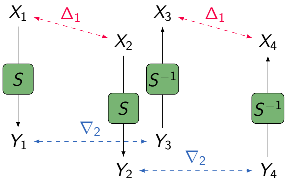




$$\mathrm{BCT}(\Delta_1, \nabla_2) \coloneqq \#\{X \in \mathbb{F}_2^n \mid S^{-1}(S(X) \oplus \nabla_2) \oplus S^{-1}(S(X \oplus \Delta_1) \oplus \nabla_2) = \Delta_1\}$$

$$\mathbb{P}(\Delta_1 \rightleftarrows \nabla_2) = 2^{-n} \cdot \mathrm{BCT}(\Delta_1, \nabla_2)$$

Generalized BCT Framework - I

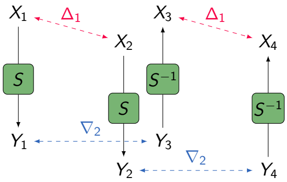


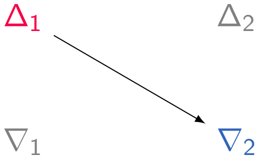
$$\Delta_1 \longrightarrow \Delta_2$$

$$\nabla_1 \longleftarrow \nabla_2$$

- $\mathcal{X}_{\mathtt{DDT}}(\Delta_1, \Delta_2) = \{x : S(x) \oplus S(x \oplus \Delta_1) = \Delta_2\}, \quad \mathtt{DDT}(\Delta_1, \Delta_2) = \#\mathcal{X}_{\mathtt{DDT}}(\Delta_1, \Delta_2)$
- $\mathcal{X}_{\mathtt{BCT}}(\Delta_1, \nabla_2) = \{x : S^{-1}(S(x) \oplus \nabla_2) \oplus S^{-1}(S(x \oplus \Delta_1) \oplus \nabla_2) = \Delta_1\}, \ \mathtt{BCT}(\Delta_1, \nabla_2) = \#\mathcal{X}_{\mathtt{BCT}}(\Delta_1, \nabla_2)$
- $\mathtt{UBCT}(\Delta_1, \Delta_2, \nabla_2) = \#\{x : x \in \mathcal{X}_{\mathtt{BCT}}(\Delta_1, \nabla_2) \cap \mathcal{X}_{\mathtt{DDT}}(\Delta_1, \Delta_2)\}$ [WP19]
- $\mathtt{LBCT}(\Delta_1, \nabla_1, \nabla_2) = \#\{x : x \in \mathcal{X}_{\mathtt{BCT}}(\Delta_1, \nabla_2) \cap \mathcal{X}_{\mathtt{DDT}}(\nabla_1, \nabla_2)\}$ [DDV20; SQH19]
- $\mathtt{EBCT}(\Delta_1, \Delta_2, \nabla_1, \nabla_2) = \#\{x : x \in \mathcal{X}_{\mathtt{BCT}}(\Delta_1, \nabla_2) \cap \mathcal{X}_{\mathtt{DDT}}(\Delta_1, \Delta_2) \cap \mathcal{X}_{\mathtt{DDT}}(\nabla_1, \nabla_2)\}$ [Bou+20; DDV20]

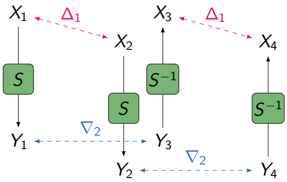
Generalized BCT Framework - I

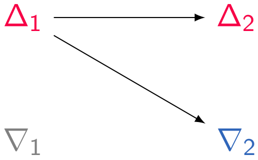




- $\mathcal{X}_{\mathtt{DDT}}(\Delta_1, \Delta_2) = \{x : S(x) \oplus S(x \oplus \Delta_1) = \Delta_2\}, \quad \mathtt{DDT}(\Delta_1, \Delta_2) = \#\mathcal{X}_{\mathtt{DDT}}(\Delta_1, \Delta_2)$
- $\mathcal{X}_{\mathtt{BCT}}(\Delta_1, \nabla_2) = \{x : S^{-1}(S(x) \oplus \nabla_2) \oplus S^{-1}(S(x \oplus \Delta_1) \oplus \nabla_2) = \Delta_1\}, \ \mathtt{BCT}(\Delta_1, \nabla_2) = \#\mathcal{X}_{\mathtt{BCT}}(\Delta_1, \nabla_2)$
- $\mathtt{UBCT}(\Delta_1, \Delta_2, \nabla_2) = \#\{x : x \in \mathcal{X}_{\mathtt{BCT}}(\Delta_1, \nabla_2) \cap \mathcal{X}_{\mathtt{DDT}}(\Delta_1, \Delta_2)\}$ [WP19]
- $\mathtt{LBCT}(\Delta_1, \nabla_1, \nabla_2) = \#\{x : x \in \mathcal{X}_{\mathtt{BCT}}(\Delta_1, \nabla_2) \cap \mathcal{X}_{\mathtt{DDT}}(\nabla_1, \nabla_2)\}$ [DDV20; SQH19]
- $\mathtt{EBCT}(\Delta_1, \Delta_2, \nabla_1, \nabla_2) = \#\{x : x \in \mathcal{X}_{\mathtt{BCT}}(\Delta_1, \nabla_2) \cap \mathcal{X}_{\mathtt{DDT}}(\Delta_1, \Delta_2) \cap \mathcal{X}_{\mathtt{DDT}}(\nabla_1, \nabla_2)\}$ [Bou+20; DDV20]

Generalized BCT Framework - I





- $\mathcal{X}_{\texttt{DDT}}(\Delta_1, \Delta_2) = \{x : S(x) \oplus S(x \oplus \Delta_1) = \Delta_2\}, \quad \texttt{DDT}(\Delta_1, \Delta_2) = \#\mathcal{X}_{\texttt{DDT}}(\Delta_1, \Delta_2)$
- $\mathcal{X}_{\texttt{BCT}}(\Delta_1, \nabla_2) = \{x : S^{-1}(S(x) \oplus \nabla_2) \oplus S^{-1}(S(x \oplus \Delta_1) \oplus \nabla_2) = \Delta_1\}, \ \texttt{BCT}(\Delta_1, \nabla_2) = \#\mathcal{X}_{\texttt{BCT}}(\Delta_1, \nabla_2)$
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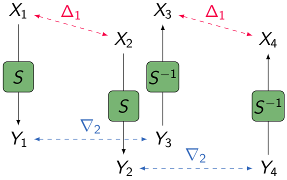
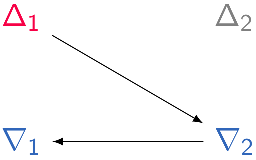
- $\texttt{EBCT}(\Delta_1, \Delta_2, \nabla_1, \nabla_2) = \#\{x : x \in \mathcal{X}_{\texttt{BCT}}(\Delta_1, \nabla_2) \cap \mathcal{X}_{\texttt{DDT}}(\Delta_1, \Delta_2) \cap \mathcal{X}_{\texttt{DDT}}(\nabla_1, \nabla_2)\}$ [Bou+20; DDV20]

Generalized BCT Framework - I





- $\mathcal{X}_{\texttt{DDT}}(\Delta_1, \Delta_2) = \{x : S(x) \oplus S(x \oplus \Delta_1) = \Delta_2\}, \quad \texttt{DDT}(\Delta_1, \Delta_2) = \#\mathcal{X}_{\texttt{DDT}}(\Delta_1, \Delta_2)$
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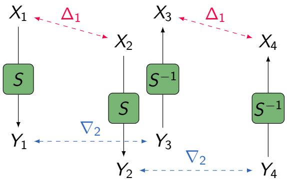
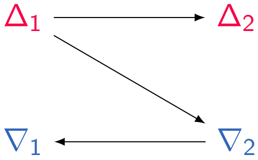
- $\texttt{LBCT}(\Delta_1, \nabla_1, \nabla_2) = \#\{x : x \in \mathcal{X}_{\texttt{BCT}}(\Delta_1, \nabla_2) \cap \mathcal{X}_{\texttt{DDT}}(\nabla_1, \nabla_2)\}$ [DDV20; SQH19]
- $\texttt{EBCT}(\Delta_1, \Delta_2, \nabla_1, \nabla_2) = \#\{x : x \in \mathcal{X}_{\texttt{BCT}}(\Delta_1, \nabla_2) \cap \mathcal{X}_{\texttt{DDT}}(\Delta_1, \Delta_2) \cap \mathcal{X}_{\texttt{DDT}}(\nabla_1, \nabla_2)\}$ [Bou+20; DDV20]

Generalized BCT Framework - I





- $\mathcal{X}_{\mathtt{DDT}}(\Delta_1, \Delta_2) = \{x : S(x) \oplus S(x \oplus \Delta_1) = \Delta_2\}, \quad \mathtt{DDT}(\Delta_1, \Delta_2) = \#\mathcal{X}_{\mathtt{DDT}}(\Delta_1, \Delta_2)$
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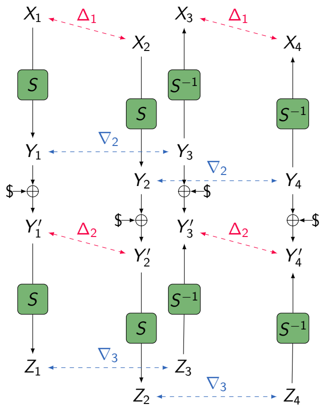
- $\mathtt{EBCT}(\Delta_1, \Delta_2, \nabla_1, \nabla_2) = \#\{x : x \in \mathcal{X}_{\mathtt{BCT}}(\Delta_1, \nabla_2) \cap \mathcal{X}_{\mathtt{DDT}}(\Delta_1, \Delta_2) \cap \mathcal{X}_{\mathtt{DDT}}(\nabla_1, \nabla_2)\}$ [Bou+20; DDV20]

Generalized BCT Framework (GBCT) - II

- Double Boomerang Connectivity Table ($\texttt{DBCT}$) [HB21]

$$\begin{tikzcd}
\Delta_1 \arrow[r] \arrow[dr] & \Delta_2 \arrow[dr] & \\
& \nabla_2^* \arrow[r] & \nabla_3
\end{tikzcd}$$



✓ $\texttt{DBCT}^{\vdash}(\Delta_1, \Delta_2, \nabla_3) = \sum_{\nabla_2} \texttt{UBCT}(\Delta_1, \Delta_2, \nabla_2) \cdot \texttt{LBCT}(\Delta_2, \nabla_2, \nabla_3)$

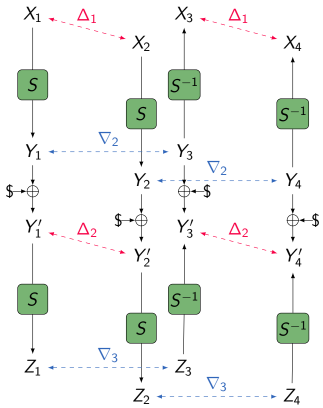
✓ $\texttt{DBCT}^{\dashv}(\Delta_1, \nabla_2, \nabla_3) = \sum_{\Delta_2} \texttt{UBCT}(\Delta_1, \Delta_2, \nabla_2) \cdot \texttt{LBCT}(\Delta_2, \nabla_2, \nabla_3)$.

✓ $\texttt{DBCT}(\Delta_1, \nabla_3) = \sum_{\Delta_2} \texttt{DBCT}^{\vdash}(\Delta_1, \Delta_2, \nabla_3) = \sum_{\nabla_2} \texttt{DBCT}^{\dashv}(\Delta_1, \nabla_2, \nabla_3)$.

Generalized BCT Framework (GBCT) - II

- Double Boomerang Connectivity Table (DBCT) [HB21]

$$
\begin{tikzcd}
\Delta_1 \arrow[r] \arrow[dr] & \Delta_2^* \arrow[dr] & \\
& \nabla_2 \arrow[r] & \nabla_3
\end{tikzcd}
$$



- $\mathtt{DBCT}^{\vdash}(\Delta_1, \Delta_2, \nabla_3) = \sum_{\nabla_2} \mathtt{UBCT}(\Delta_1, \Delta_2, \nabla_2) \cdot \mathtt{LBCT}(\Delta_2, \nabla_2, \nabla_3)$
- $\mathtt{DBCT}^{\dashv}(\Delta_1, \nabla_2, \nabla_3) = \sum_{\Delta_2} \mathtt{UBCT}(\Delta_1, \Delta_2, \nabla_2) \cdot \mathtt{LBCT}(\Delta_2, \nabla_2, \nabla_3).$
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ASK 2024 - Kolkata, India

Generalized BCT Framework (GBCT) - II

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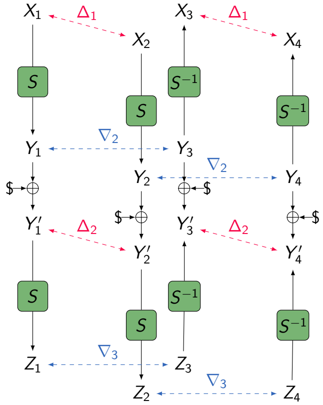

- $\mathtt{DBCT}^{\vdash}(\Delta_1, \Delta_2, \nabla_3) = \sum_{\nabla_2} \mathtt{UBCT}(\Delta_1, \Delta_2, \nabla_2) \cdot \mathtt{LBCT}(\Delta_2, \nabla_2, \nabla_3)$
- $\mathtt{DBCT}^{\dashv}(\Delta_1, \nabla_2, \nabla_3) = \sum_{\Delta_2} \mathtt{UBCT}(\Delta_1, \Delta_2, \nabla_2) \cdot \mathtt{LBCT}(\Delta_2, \nabla_2, \nabla_3).$
- $\mathtt{DBCT}(\Delta_1, \nabla_3) = \sum_{\Delta_2} \mathtt{DBCT}^{\vdash}(\Delta_1, \Delta_2, \nabla_3) = \sum_{\nabla_2} \mathtt{DBCT}^{\dashv}(\Delta_1, \nabla_2, \nabla_3).$

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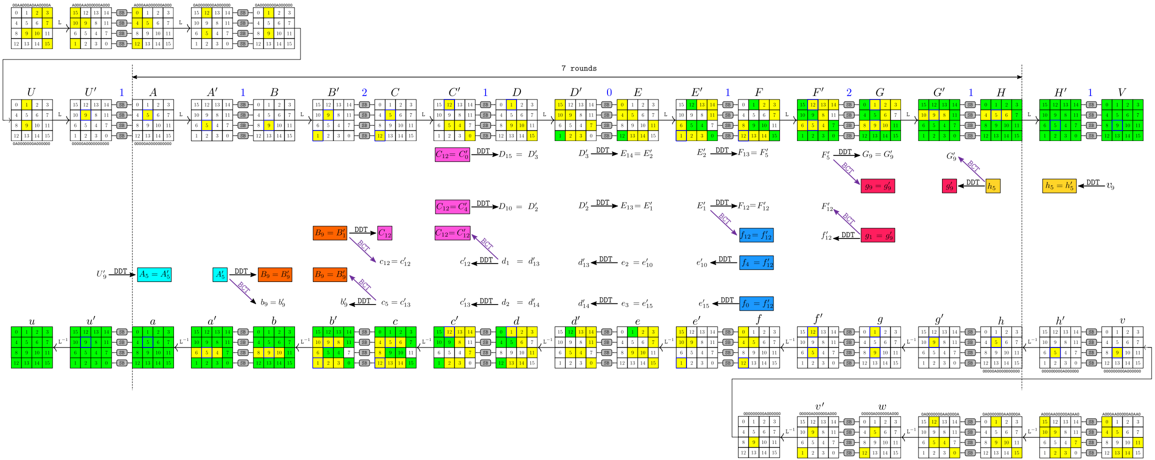
Application of GBCT [HB21]



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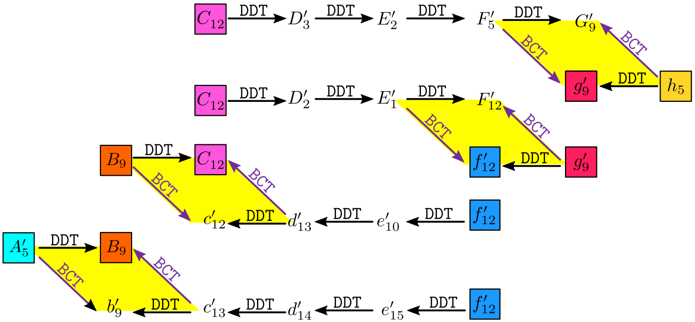
Application of GBCT [HB21]



$$\mathtt{DBCT}_{\mathrm{total}} = \mathtt{DBCT}^{\vdash}(A_5, B_9, c_5) \cdot \mathtt{DBCT}^{\vdash}(B_9, C_{12}, d_1) \cdot \mathtt{DBCT}^{\dashv}(E'_1, f'_{12}, g'_9) \cdot \mathtt{DBCT}^{\dashv}(F'_5, g'_9, h_5)$$

$$\mathrm{Pr}_{\mathrm{total}} = \mathrm{Pr}(d_1 \xleftarrow{2\ \mathtt{DDT}} f'_{12}) \cdot \mathrm{Pr}(c_5 \xleftarrow{3\ \mathtt{DDT}} f'_{12}) \cdot \mathrm{Pr}(C_{12} \xrightarrow{2\ \mathtt{DDT}} E'_1) \cdot \mathrm{Pr}(C_{12} \xrightarrow{3\ \mathtt{DDT}} F'_5)$$

$$r = 2^{-8 \cdot n} \cdot \sum_{B_9} \sum_{C_{12}} \sum_{g'_9} \sum_{f'_{12}} \sum_{c_5} \sum_{d_1} \sum_{E'_1} \sum_{F'_5} \mathtt{DBCT}_{\mathrm{total}} \cdot \mathrm{Pr}_{\mathrm{total}}.$$

Differential-Linear (DL) Attack I [LH94]

Input: $E_K, (\Delta, \lambda), N, \boldsymbol{c} = \mathbb{C}(\Delta, \lambda)$

Output: 0: **real** cipher, 1: **ideal** cipher

Initialize a counter list $V[z] \leftarrow 0$ for $z \in \{0, 1\}$;

for $i = 0, \ldots, N-1$ **do**

  $P_1 \xleftarrow{\$} \mathbb{F}_2^n$;

  $b_1 \leftarrow \lambda \cdot E_K(P_1)$;

  $P_2 \leftarrow P_1 \oplus \Delta$;

  $b_2 \leftarrow \lambda \cdot E_K(P_2)$;

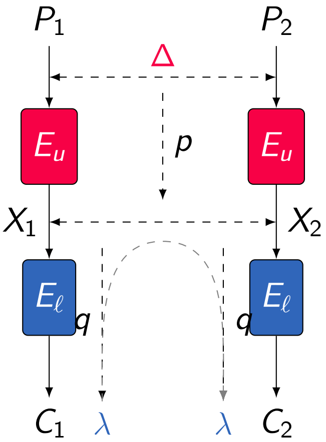
  $V[b_1 \oplus b_2] \leftarrow V[b_1 \oplus b_2] + 1$;

if $V[0] \sim \mathcal{N}(\mu = N\frac{1+\boldsymbol{c}}{2}, \sigma^2 = N\frac{1-\boldsymbol{c}^2}{4})$ **then**

  return 0; `// real cipher`

else

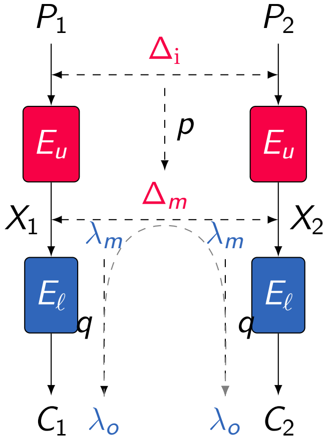
  return 1; `// ideal cipher`



$\boldsymbol{c} = 2 \cdot \mathbb{P}(\boldsymbol{\lambda} \cdot C_1 \oplus \boldsymbol{\lambda} \cdot C_2 = 0) - 1$

Differential-Linear (DL) Attack II [LH94]

- $p = \mathbb{P}(\Delta_{\rm i} \xrightarrow{E_u} \Delta_m)$
- $q = \mathbb{C}(\lambda_m \xrightarrow{E_\ell} \lambda_{\rm o}) = 2 \cdot \mathbb{P}(\lambda_m \cdot X \oplus \lambda_{\rm o} \cdot E_\ell(X) = 0) - 1$
- Assumptions ($\Delta X = X_1 \oplus X_2$):
  1. E_u, and E_ℓ are statistically independent
  2. $\mathbb{P}(\lambda_m \cdot \Delta X = 0) = 1/2$ when $\Delta X \neq \Delta_m$
- $\mathcal{C} = \mathbb{C}(\lambda_{\rm o} \cdot \Delta C) \approx (-1)^{\lambda_m \cdot \Delta_m} \cdot pq^2 = \pm pq^2$
- Time/Data complexity: $\mathcal{O}(\mathcal{C}^{-2})$



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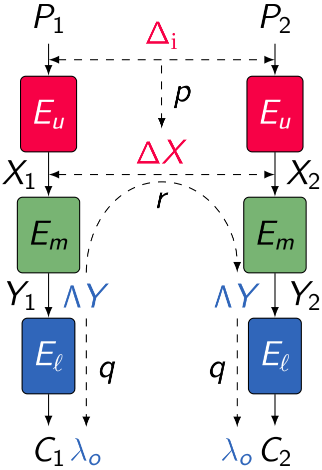
Sandwich Framework for DL Attack [BLN14; DKS14; Bar+19]

- $\mathbb{R}(\Delta X, \Lambda Y) = \mathbb{C}\left(\Lambda Y \cdot E_m(X) \oplus \Lambda Y \cdot E_m(X \oplus \Delta X)\right)$
- $\mathbb{C}(\lambda_{\mathrm{o}} \cdot \Delta C) = \sum_{\Delta X, \Lambda Y} \mathbb{P}(\Delta_{\mathrm{i}}, \Delta X) \cdot \mathbb{R}(\Delta X, \Lambda Y) \cdot \mathbb{C}^2(\Lambda Y, \lambda_{\mathrm{o}})$
- $\mathbb{P}(\Delta_{\mathrm{i}} \xrightarrow{E_u} \Delta_m) = p$
- $\mathbb{R}(\Delta_m, \lambda_m) = r$
- $\mathbb{C}(\lambda_m \xrightarrow{E_\ell} \lambda_{\mathrm{o}}) = q$
- $\mathbb{C}(\lambda_{\mathrm{o}} \cdot \Delta C) \approx prq^2$



Sandwich Framework for DL Attack [BLN14; DKS14; Bar+19]

- $\mathbb{R}(\Delta X, \Lambda Y) = \mathbb{C}\left(\Lambda Y \cdot E_m(X) \oplus \Lambda Y \cdot E_m(X \oplus \Delta X)\right)$
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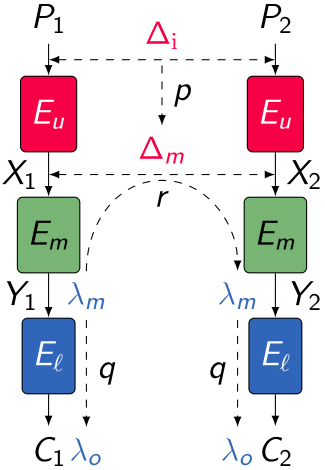
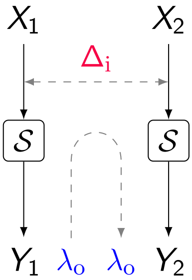
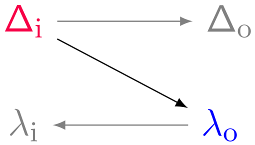
- $\mathbb{P}(\Delta_\mathrm{i} \xrightarrow{E_u} \Delta_m) = p$
- $\mathbb{R}(\Delta_m, \lambda_m) = r$
- $\mathbb{C}(\lambda_m \xrightarrow{E_\ell} \lambda_\mathrm{o}) = q$
- $\mathbb{C}(\lambda_\mathrm{o} \cdot \Delta C) \approx prq^2$



Differential-Linear Connectivity Table (DLCT) [Bar+19]



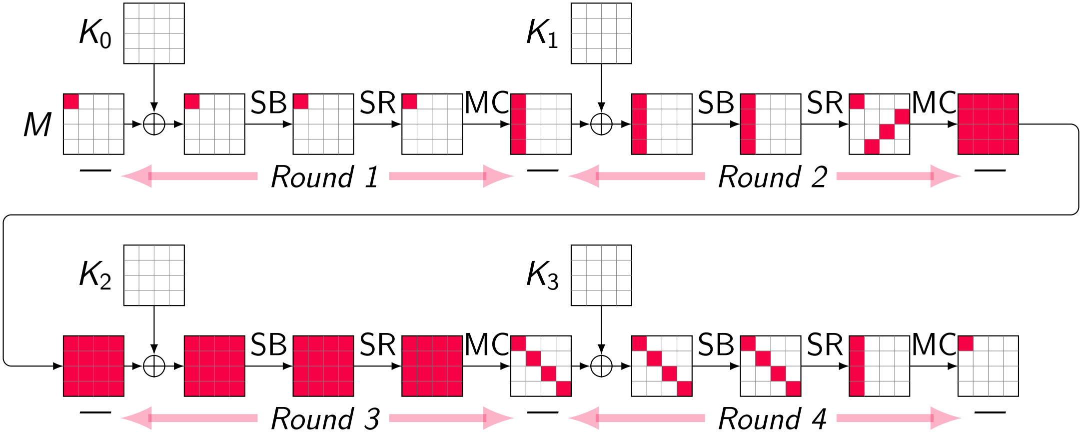


$$\mathtt{DLCT}_b(\Delta_\mathrm{i}, \lambda_\mathrm{o}) = \{x \in \mathbb{F}_2^n : \ \lambda_\mathrm{o} \cdot S(x) \oplus \lambda_\mathrm{o} \cdot S(x \oplus \Delta_\mathrm{i}) = b\}$$

$$\mathtt{DLCT}(\Delta_\mathrm{i}, \lambda_\mathrm{o}) = |\mathtt{DLCT}_0(\Delta_\mathrm{i}, \lambda_\mathrm{o})| - |\mathtt{DLCT}_1(\Delta_\mathrm{i}, \lambda_\mathrm{o})|$$

$$\mathbb{C}_{\mathtt{DLCT}}(\Delta_\mathrm{i}, \lambda_\mathrm{o}) = 2^{-n} \cdot \mathtt{DLCT}(\Delta_\mathrm{i}, \lambda_\mathrm{o})$$

Security of AES Against Differential/Linear Attacks



$$\mathbb{P}_{4\text{ rounds}} \leq 2^{-150},\ \mathbb{C}^2_{4\text{ rounds}} \leq 2^{-150}$$

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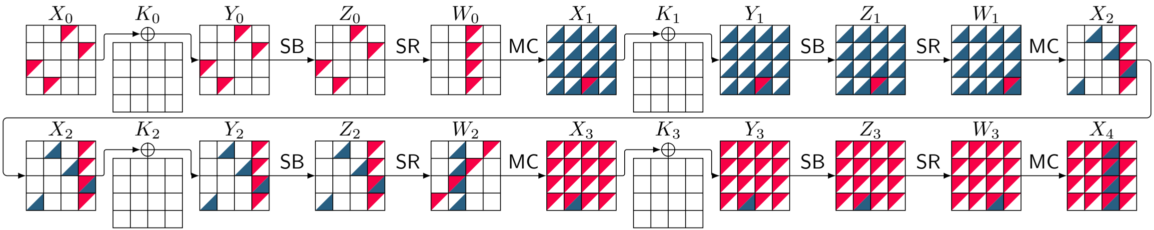
A 4-round DL Distinguisher for AES

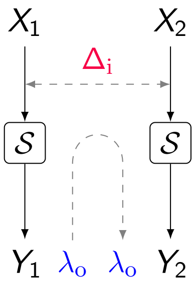


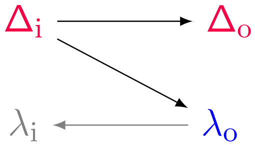
$r_u = 1, r_m = 3, r_\ell = 0,\ \ p = 2^{-24.00},\ \ r = 2^{-7.66}, q^2 = 1,\ \ \mathbb{C} = prq^2 = \mathbf{2^{-31.66}}$	
ΔX_0 `00005200000000f58f000000007b0000`	ΔX_1 `000000000000000000000000000b400`
ΓX_4 `0032000000ab00000066000000980000`	-

$$\mathbf{2^{63.32}} \text{ v.s. } 2^{150}$$

Generalized DLCT Framework

Upper Differential-Linear Connectivity Table (UDLCT)





$$\mathtt{UDLCT}_b(\Delta_i, \Delta_o, \lambda_o) = \{x \in \mathbb{F}_2^n : \; S(x) \oplus S(x \oplus \Delta_i) = \Delta_o \text{ and } \lambda_o \cdot \Delta_o = b\}$$

$$\mathtt{UDLCT}(\Delta_i, \Delta_o, \lambda_o) = |\mathtt{UDLCT}_0(\Delta_i, \Delta_o, \lambda_o)| - |\mathtt{UDLCT}_1(\Delta_i, \Delta_o, \lambda_o)|$$

$$\mathbb{C}_{\mathtt{UDLCT}}(\Delta_i, \Delta_o, \lambda_o) = 2^{-n} \cdot \mathtt{UDLCT}(\Delta_i, \Delta_o, \lambda_o)$$

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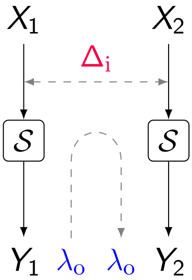
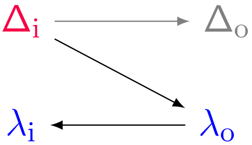
Lower Differential-Linear Connectivity Table ($\mathtt{LDLCT}$)



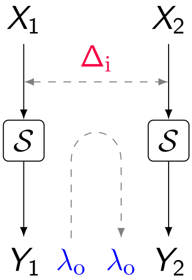


$$\mathtt{LDLCT}_b(\Delta_\mathrm{i}, \lambda_\mathrm{i}, \lambda_\mathrm{o}) = \{x \in \mathbb{F}_2^n : \ \lambda_\mathrm{i} \cdot \Delta_\mathrm{i} \oplus \lambda_\mathrm{o} \cdot S(x) \oplus \lambda_\mathrm{o} \cdot S(x \oplus \Delta_\mathrm{i}) = b\}$$

$$\mathtt{LDLCT}(\Delta_\mathrm{i}, \lambda_\mathrm{i}, \lambda_\mathrm{o}) = |\mathtt{LDLCT}_0(\Delta_\mathrm{i}, \lambda_\mathrm{i}, \lambda_\mathrm{o})| - |\mathtt{LDLCT}_1(\Delta_\mathrm{i}, \lambda_\mathrm{i}, \lambda_\mathrm{o})|$$

$$\mathbb{C}_{\mathtt{LDLCT}}\left(\Delta_\mathrm{i}, \lambda_\mathrm{i}, \lambda_\mathrm{o}\right) = 2^{-n} \cdot \mathtt{LDLCT}\left(\Delta_\mathrm{i}, \lambda_\mathrm{i}, \lambda_\mathrm{o}\right)$$

Extended Differential-Linear Connectivity Table (EDLCT)



$$\begin{tikzcd} \Delta_i \arrow[r] \arrow[dr] & \Delta_o \\ \lambda_i & \lambda_o \arrow[l] \end{tikzcd}$$

$$\texttt{EDLCT}_b(\Delta_i, \Delta_o, \lambda_i, \lambda_o) = \{x \in \mathbb{F}_2^n : \ S(x) \oplus S(x \oplus \Delta_i) = \Delta_o \text{ and } \lambda_i \cdot \Delta_i \oplus \lambda_o \cdot \Delta_o = b\}$$

$$\texttt{EDLCT}(\Delta_i, \Delta_o, \lambda_i, \lambda_o) = |\texttt{EDLCT}_0(\Delta_i,\Delta_o,\lambda_i,\lambda_o)| - |\texttt{EDLCT}_1(\Delta_i,\Delta_o,\lambda_i,\lambda_o)|$$

$$\mathbb{C}_{\texttt{EDLCT}}(\Delta_i, \Delta_o, \lambda_i, \lambda_o) = 2^{-n} \cdot \texttt{EDLCT}(\Delta_i, \Delta_o, \lambda_i, \lambda_o)$$

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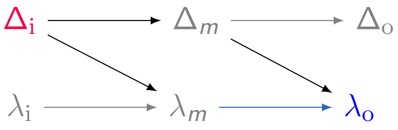
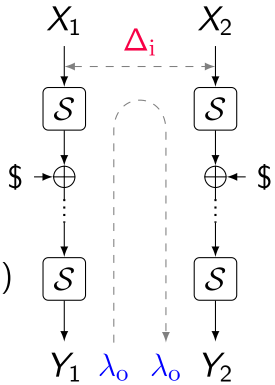
Double Differential-Linear Connectivity Table (DDLCT)

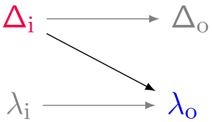


$$\mathtt{DDLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = 2^{-n} \cdot \sum_{\Delta_m} \sum_{\lambda_m} \mathtt{UDLCT}\left(\Delta_{\mathrm{i}}, \Delta_m, \lambda_m\right) \cdot \mathtt{LDLCT}\left(\Delta_m, \lambda_m, \lambda_{\mathrm{o}}\right)$$

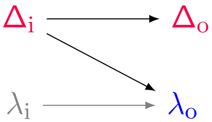


Generalized DLCT Framework (GBCT)

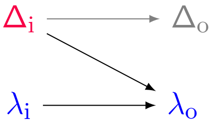
- How to formulate the correaltion for more than 1 round?



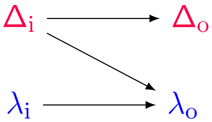
$\mathtt{DLCT}\left(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}\right)$



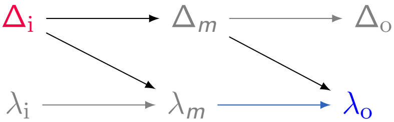
$\mathtt{UDLCT}\left(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{o}}\right)$



$\mathtt{LDLCT}\left(\Delta_{\mathrm{i}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}}\right)$

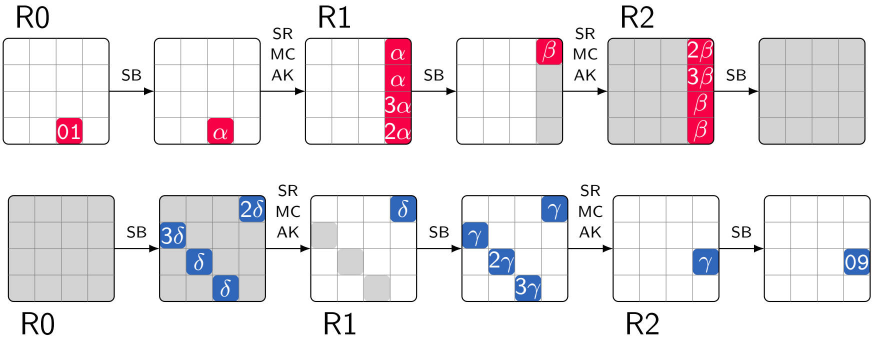


$\mathtt{EDLCT}\left(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}}\right)$



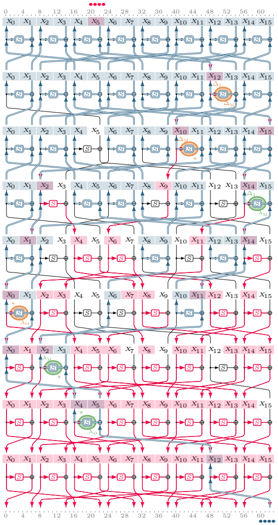
$\mathtt{DDLCT}\left(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}\right)$

Application of the Generalized DLCT Tables - AES (- differential - linear)



$$\sum_{\alpha,\beta,\gamma,\delta} \mathbb{C}_{\mathtt{UDLCT}}(1, \alpha, \delta) \cdot \mathbb{C}_{\mathtt{EDLCT}}(\alpha, \beta, \delta, \gamma) \cdot \mathbb{C}_{\mathtt{LDLCT}}(\beta, \gamma, 9) = -2^{-7.94}$$

Application of the Generalized DLCT Tables - TWINE (- differential - linear)



$$\mathbb{C}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = \sum_{\Delta_m} \mathbb{P}_{\mathtt{DDT}}(\Delta_{\mathrm{i}}, \Delta_m) \cdot \mathbb{C}_{\mathtt{DDLCT}}(\Delta_m, \lambda_{\mathrm{o}})$$

$$= \sum_{\lambda_m} \mathbb{C}_{\mathtt{DDLCT}}(\Delta_{\mathrm{i}}, \lambda_m) \cdot \mathbb{C}^2_{\mathtt{LAT}}(\lambda_m, \lambda_{\mathrm{o}}) .$$

$$\mathbb{C}_{tot}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = \mathbb{C}^2(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}).$$

Input/Output Differences/Linear-mask	Formula	Exp. Correlation
$(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = (\mathtt{0xb4}, \mathtt{0x67})$	$-2^{-7.66}$	$-2^{-7.64}$
$(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = (\mathtt{0x02}, \mathtt{0x02})$	$-2^{-7.92}$	$-2^{-7.93}$
$(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = (\mathtt{0x55}, \mathtt{0x55})$	$-2^{-7.99}$	$-2^{-7.98}$
$(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = (\mathtt{0xbf}, \mathtt{0xef})$	$-2^{-8.05}$	$-2^{-8.06}$
$(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = (\mathtt{0xfe}, \mathtt{0x06})$	$-2^{-8.26}$	$-2^{-8.25}$
$(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = (\mathtt{0x4b}, \mathtt{0x1a})$	$-2^{-8.43}$	$-2^{-8.44}$

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Differential-Linear Switches and Deterministic Trails

Cell-Wise and Bit-Wise Switches

x	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$\mathcal{S}(x)$	4	0	a	7	b	e	1	d	9	f	6	8	5	2	c	3

$\Delta \setminus \lambda$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
1	16	0	0	0	-16	0	0	0	0	0	0	0	0	0	0	0
2	16	-8	-8	0	0	0	8	-8	0	-8	0	8	0	0	0	0
3	16	0	-8	-8	0	-8	8	0	0	0	0	0	0	-8	0	8
4	16	0	-8	0	0	0	-8	0	-16	0	8	0	0	0	8	0
5	16	0	-8	0	0	0	-8	0	0	0	8	0	-16	0	8	0
6	16	-8	8	-8	0	0	-8	0	0	-8	0	0	0	0	0	8
7	16	0	8	0	0	-8	-8	-8	0	0	0	8	0	-8	0	0
8	16	0	0	0	-16	0	0	0	-16	0	0	0	16	0	0	0
9	16	-8	0	-8	16	-8	0	-8	0	8	0	-8	0	8	0	-8
a	16	0	0	8	0	8	0	0	0	0	-8	0	0	-8	-8	-8
b	16	8	0	0	0	0	0	8	0	-8	-8	-8	0	0	-8	0
c	16	0	0	-8	0	0	0	-8	16	0	0	-8	0	0	0	-8
d	16	-8	0	0	0	-8	0	0	0	8	0	0	-16	8	0	0
e	16	0	0	0	0	8	0	8	0	0	-8	-8	0	-8	-8	0
f	16	8	0	8	0	0	0	0	0	-8	-8	0	0	0	-8	-8

- Cell-wise switches: $\mathtt{DLCT}(\Delta_{\mathrm{i}}, 0) = \mathtt{DLCT}(0, \lambda_{\mathrm{o}}) = 2^n$ for all $\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}$
- Bit-wise switches: $\mathtt{DLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = \pm 2^n$ for $\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}} \neq 0$
  - Example: $\mathbb{C}\,(9, 4) = \frac{16}{16}$

Deterministic Bit-Wise Differential Trails (Forward)

x	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$\mathcal{S}(x)$	4	0	a	7	b	e	1	d	9	f	6	8	5	2	c	3

$\Delta_i \setminus \Delta_o$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	2	2	2	2	0	0	0	0	2	2	2	2
2	0	2	0	2	0	0	0	4	0	2	2	0	0	0	2	2
3	0	2	0	2	0	0	4	0	0	2	2	0	0	0	2	2
4	0	0	0	0	0	0	0	0	0	0	4	4	2	2	2	2
5	0	0	0	0	2	2	2	2	0	0	4	4	0	0	0	0
6	0	2	0	2	0	4	0	0	0	2	2	0	2	2	0	0
7	0	2	0	2	4	0	0	0	0	2	2	0	2	2	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	4	4	4	4
9	0	4	4	0	0	0	0	0	0	4	0	4	0	0	0	0
a	0	0	2	2	2	0	0	2	4	0	0	0	0	2	0	2
b	0	0	2	2	0	2	2	0	4	0	0	0	2	0	2	0
c	0	4	4	0	2	2	2	2	0	0	0	0	0	0	0	0
d	0	0	0	0	2	2	2	2	0	4	0	4	0	0	0	0
e	0	0	2	2	0	2	2	0	4	0	0	0	0	2	0	2
f	0	0	2	2	2	0	0	2	4	0	0	0	2	0	2	0

$$\Delta_i = (0,0,0,0) \xrightarrow{S} \Delta_o = (0,0,0,0)$$

$$\Delta_i = (0,0,0,1) \xrightarrow{S} \Delta_o = (?,1,?,?)$$

$$\Delta_i = (0,1,0,0) \xrightarrow{S} \Delta_o = (1,?,?,?)$$

$$\Delta_i = (1,0,0,0) \xrightarrow{S} \Delta_o = (1,1,?,?)$$

$$\Delta_i = (1,0,0,1) \xrightarrow{S} \Delta_o = (?,0,?,?)$$

$$\Delta_i = (1,1,0,0) \xrightarrow{S} \Delta_o = (0,?,?,?)$$

Deterministic Bit-Wise Linear Trails (Backward)

x	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$\mathcal{S}(x)$	4	0	a	7	b	e	1	d	9	f	6	8	5	2	c	3

$\lambda_i \setminus \lambda_o$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	4	-4	0	-8	-4	-4	0	0	4	-4	-8	0	4	4
2	0	0	0	0	0	0	0	0	0	8	8	0	0	8	-8	0
3	0	-8	4	4	0	0	-4	4	0	0	-4	4	-8	0	-4	-4
4	0	4	0	4	0	4	8	-4	0	4	0	4	-8	-4	0	4
5	0	4	-4	-8	0	-4	-4	0	0	4	-4	8	0	-4	-4	0
6	0	-4	8	4	0	-4	0	-4	0	4	0	4	8	-4	0	4
7	0	4	4	0	0	-4	4	-8	0	-4	-4	0	0	4	-4	-8
8	0	0	0	0	0	0	0	0	0	0	8	8	0	0	8	-8
9	0	0	-4	4	8	0	-4	-4	0	0	4	-4	0	-8	-4	-4
a	0	8	0	8	0	-8	0	8	0	0	0	0	0	0	0	0
b	0	0	-4	4	-8	0	-4	-4	0	8	-4	-4	0	0	4	-4
c	0	4	0	4	0	4	-8	-4	8	-4	0	4	0	4	0	4
d	0	4	4	0	-8	4	-4	0	-8	-4	4	0	0	-4	-4	0
e	0	4	8	-4	0	4	0	4	8	4	0	-4	0	-4	0	-4
f	0	-4	-4	0	-8	-4	4	0	8	-4	4	0	0	-4	-4	0

$$\lambda_{\mathrm{i}} = (1, ?, ?, 1) \xleftarrow{S} \lambda_{\mathrm{o}} = (0, 1, 0, 0)$$

$$\lambda_{\mathrm{i}} = (1, 1, ?, ?) \xleftarrow{S} \lambda_{\mathrm{o}} = (1, 0, 0, 0)$$

$$\lambda_{\mathrm{i}} = (0, ?, ?, ?) \xleftarrow{S} \lambda_{\mathrm{o}} = (1, 1, 0, 0)$$

Bit-Wise Switches and Deterministic Trails

x	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$\mathcal{S}(x)$	4	0	a	7	b	e	1	d	9	f	6	8	5	2	c	3

$\Delta \setminus \lambda$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
1	16	0	0	0	-16	0	0	0	0	0	0	0	0	0	0	0
2	16	-8	-8	0	0	0	8	-8	0	-8	0	8	0	0	0	0
3	16	0	-8	-8	0	-8	8	0	0	0	0	0	0	-8	0	8
4	16	0	-8	0	0	0	-8	0	-16	0	8	0	0	0	8	0
5	16	0	-8	0	0	0	-8	0	0	0	8	0	-16	0	8	0
6	16	-8	8	-8	0	0	-8	0	0	-8	0	0	0	0	0	8
7	16	0	8	0	0	-8	-8	-8	0	0	0	8	0	-8	0	0
8	16	0	0	0	-16	0	0	0	-16	0	0	0	16	0	0	0
9	16	-8	0	-8	16	-8	0	-8	0	8	0	-8	0	8	0	-8
a	16	0	0	8	0	8	0	0	0	0	-8	0	0	-8	-8	-8
b	16	8	0	0	0	0	0	8	0	-8	-8	-8	0	0	-8	0
c	16	0	0	-8	0	0	0	-8	16	0	0	-8	0	0	0	-8
d	16	-8	0	0	0	-8	0	0	0	8	0	0	-16	8	0	0
e	16	0	0	0	0	8	0	8	0	0	-8	-8	0	-8	-8	0
f	16	8	0	8	0	0	0	0	0	-8	-8	0	0	0	-8	-8

$$\Delta_{\mathrm{i}} = (0,0,0,1) \xrightarrow{S} \Delta_{\mathrm{o}} = (?,1,?,?)$$

$$\Delta_{\mathrm{i}} = (0,1,0,0) \xrightarrow{S} \Delta_{\mathrm{o}} = (1,?,?,?)$$

$$\Delta_{\mathrm{i}} = (1,0,0,0) \xrightarrow{S} \Delta_{\mathrm{o}} = (1,1,?,?)$$

$$\Delta_{\mathrm{i}} = (1,0,0,1) \xrightarrow{S} \Delta_{\mathrm{o}} = (?,0,?,?)$$

$$\Delta_{\mathrm{i}} = (1,1,0,0) \xrightarrow{S} \Delta_{\mathrm{o}} = (0,?,?,?)$$

$$\lambda_{\mathrm{i}} = (1,?,?,1) \xleftarrow{S} \lambda_{\mathrm{o}} = (0,1,0,0)$$

$$\lambda_{\mathrm{i}} = (1,1,?,?) \xleftarrow{S} \lambda_{\mathrm{o}} = (1,0,0,0)$$

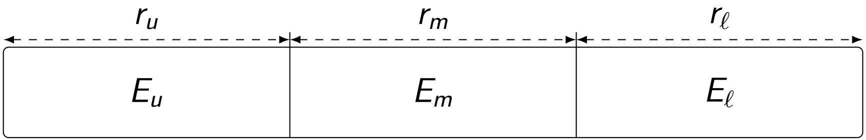
$$\lambda_{\mathrm{i}} = (0,?,?,?) \xleftarrow{S} \lambda_{\mathrm{o}} = (1,1,0,0)$$

Automatic Tools to Search for DL Distinguishers

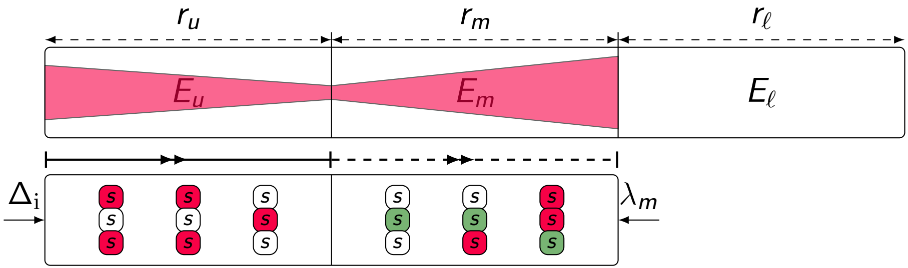
Overview of Our Method to Search for Distinguishers

E

Overview of Our Method to Search for Distinguishers

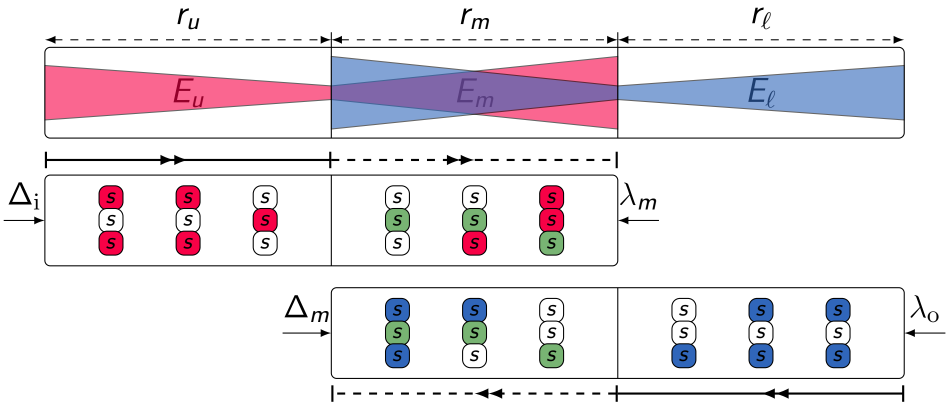


Overview of Our Method to Search for Distinguishers



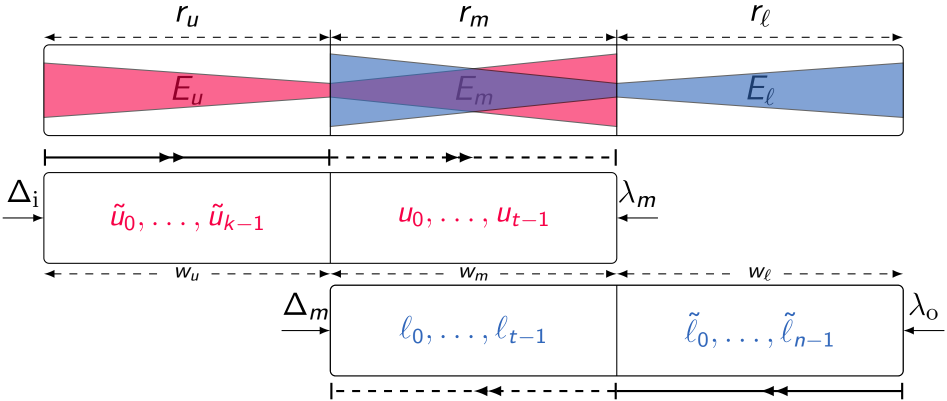
differentially active S-box   linearly active S-box   common active S-box

Overview of Our Method to Search for Distinguishers



differentially active S-box
linearly active S-box
common active S-box

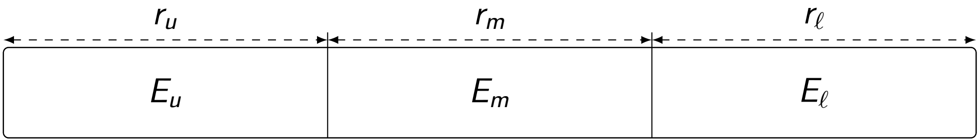
Overview of Our Method to Search for Distinguishers



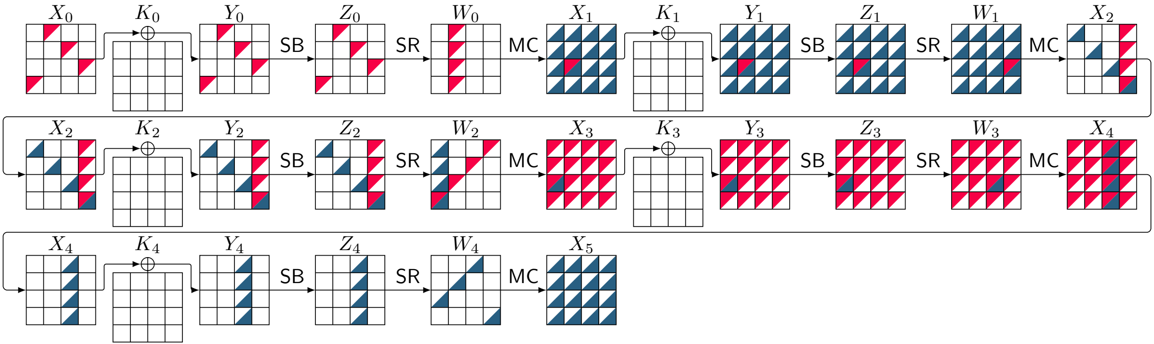
$$\min\left(\sum_{i=0}^{k-1} w_u \cdot \tilde{u}_i + \sum_{j=0}^{t-1} w_m \cdot \mathtt{bool2int}\left(\ell_j + u_j = 2\right) + \sum_{k=0}^{n-1} w_\ell \cdot \tilde{\ell}_k\right)$$

Usage of Our Tool

```
python3 attack.py -RU 6 -RM 10 -RL 6
```



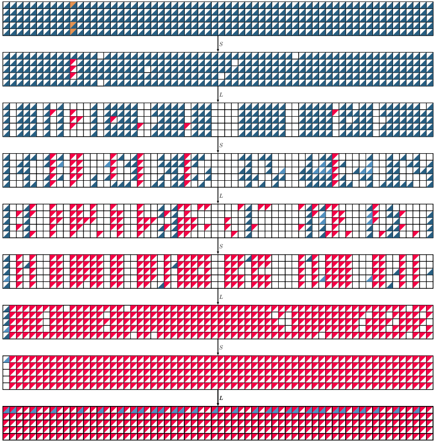
Results: A 5-round DL Distinguisher for AES



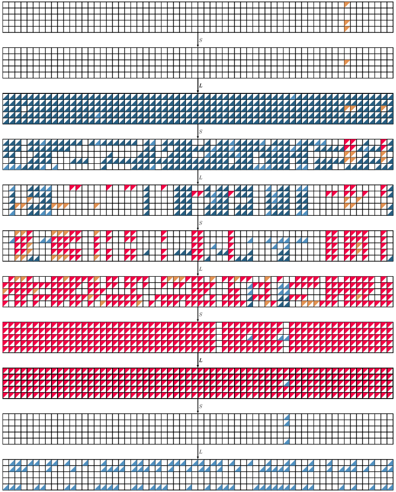
$r_0 = 1, r_m = 3, r_1 = 1, \ p = 2^{-24.00}, r = 2^{-7.66}, \ q^2 = 2^{-24.00}, \ prq^2 = 2^{-55.66}$

ΔX_0	`001c00000000e200000000dfb3000000`	ΔX_1	`00000000000000000f70000000000000`
ΓX_4	`00000000000000006700000000000000`	ΓX_5	`21d3814d93b1ef228e923507f67383fd`

Results: Application to Ascon-p(active difference unknown difference active mask unknown mask)



$\mathbb{C} = 1$



$\mathbb{C} = 2^{-4.33}$

Contributions and Future Works

Contributions and Future Works

- Contributions
  - We generalized the DLCT framework from one S-box layer to multiple rounds
  - We proposed an automatic tool for finding optimum DL distinguishers
  - We applied our tool to almost any design paradigm
- Future works
  - Extending the application of our tool to other primitives, e.g., ARX
  - Extending our tool to a unified model for finding complete attack (key recovery)

: https://github.com/hadipourh/DL

: https://ia.cr/2024/255

Bibliography I

[Bar+19] Achiya Bar-On et al. **DLCT: A New Tool for Differential-Linear Cryptanalysis**. EUROCRYPT 2019. Vol. 11476. LNCS. Springer, 2019, pp. 313–342. DOI: 10.1007/978-3-030-17653-2_11.

[BLN14] Céline Blondeau, Gregor Leander, and Kaisa Nyberg. **Differential-Linear Cryptanalysis Revisited**. FSE 2014. Ed. by Carlos Cid and Christian Rechberger. Vol. 8540. LNCS. Springer, 2014, pp. 411–430. DOI: 10.1007/978-3-662-46706-0_21.

[Bou+20] Hamid Boukerrou et al. **On the Feistel Counterpart of the Boomerang Connectivity Table Introduction and Analysis of the FBCT**. *IACR Trans. Symmetric Cryptol.* 2020.1 (2020), pp. 331–362. DOI: 10.13154/TOSC.V2020.I1.331-362.

Bibliography II

[BS90] Eli Biham and Adi Shamir. **Differential Cryptanalysis of DES-like Cryptosystems**. CRYPTO '90. Ed. by Alfred Menezes and Scott A. Vanstone. Vol. 537. LNCS. Springer, 1990, pp. 2–21. DOI: 10.1007/3-540-38424-3_1.

[Cid+18] Carlos Cid et al. **Boomerang Connectivity Table: A New Cryptanalysis Tool**. EUROCRYPT 2018. Ed. by Jesper Buus Nielsen and Vincent Rijmen. Vol. 10821. LNCS. Springer, 2018, pp. 683–714. DOI: 10.1007/978-3-319-78375-8_22.

[DDV20] Stéphanie Delaune, Patrick Derbez, and Mathieu Vavrille. **Catching the Fastest Boomerangs Application to SKINNY**. *IACR Trans. Symmetric Cryptol.* 2020.4 (2020), pp. 104–129. DOI: 10.46586/TOSC.V2020.I4.104-129.

Bibliography III

[DIK08] Orr Dunkelman, Sebastiaan Indesteege, and Nathan Keller. **A Differential-Linear Attack on 12-Round Serpent**. INDOCRYPT 2008. Ed. by Dipanwita Roy Chowdhury, Vincent Rijmen, and Abhijit Das. Vol. 5365. LNCS. Springer, 2008, pp. 308–321. DOI: 10.1007/978-3-540-89754-5_24.

[DKS10] Orr Dunkelman, Nathan Keller, and Adi Shamir. **A Practical-Time Related-Key Attack on the KASUMI Cryptosystem Used in GSM and 3G Telephony**. CRYPTO. Vol. 6223. LNCS. Springer, 2010, pp. 393–410. DOI: 10.1007/978-3-642-14623-7_21.

[DKS14] Orr Dunkelman, Nathan Keller, and Adi Shamir. **A Practical-Time Related-Key Attack on the KASUMI Cryptosystem Used in GSM and 3G Telephony**. *J. Cryptol.* 27.4 (2014), pp. 824–849. DOI: 10.1007/s00145-013-9154-9.

Bibliography IV

[HB21] Hosein Hadipour and Nasour Bagheri. **Improved Rectangle Attacks on SKINNY and CRAFT**. *IACR Trans. Symmetric Cryptol.* 2021.2 (2021), pp. 140–198. DOI: 10.46586/TOSC.V2021.I2.140-198.

[HNE22] Hosein Hadipour, Marcel Nageler, and Maria Eichlseder. **Throwing Boomerangs into Feistel Structures Application to CLEFIA, WARP, LBlock, LBlock-s and TWINE**. *IACR Trans. Symmetric Cryptol.* 2022.3 (2022), pp. 271–302. DOI: 10.46586/TOSC.V2022.I3.271-302.

[LH94] Susan K. Langford and Martin E. Hellman. **Differential-Linear Cryptanalysis**. CRYPTO '94. Vol. 839. Springer, 1994, pp. 17–25. DOI: 10.1007/3-540-48658-5_3.

Bibliography V

[Mat93] Mitsuru Matsui. **Linear Cryptanalysis Method for DES Cipher**. EUROCRYPT '93. Ed. by Tor Helleseth. Vol. 765. LNCS. Springer, 1993, pp. 386–397. DOI: 10.1007/3-540-48285-7_33.

[SQH19] Ling Song, Xianrui Qin, and Lei Hu. **Boomerang Connectivity Table Revisited. Application to SKINNY and AES**. *IACR Trans. Symmetric Cryptol.* 2019.1 (2019), pp. 118–141. DOI: 10.13154/TOSC.V2019.I1.118-141. URL: https://doi.org/10.13154/tosc.v2019.i1.118-141.

[Wag99] David A. Wagner. **The Boomerang Attack**. FSE. Vol. 1636. LNCS. Springer, 1999, pp. 156–170. DOI: 10.1007/3-540-48519-8_12.

[WP19] Haoyang Wang and Thomas Peyrin. **Boomerang Switch in Multiple Rounds. Application to AES Variants and Deoxys**. *IACR Trans. Symmetric Cryptol.* 2019.1 (2019), pp. 142–169. DOI: 10.13154/TOSC.V2019.I1.142-169.

Bibliography VI

[ZWH24] Yanyan Zhou, Senpeng Wang, and Bin Hu. **MILP/MIQCP-Based Fully Automatic Method of Searching for Differential-Linear Distinguishers for SIMON-Like Ciphers**. *IET Information Security* 2024 (2024). DOI: 10.1049/2024/8315115.

Properties of Generalized $\texttt{DLCT}$ Tables - I

- $\texttt{DLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = \sum_{\Delta_{\mathrm{o}}} \texttt{UDLCT}(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{o}})$
- $\texttt{UDLCT}(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{o}}) = (-1)^{\Delta_{\mathrm{o}} \cdot \lambda_{\mathrm{o}}} \texttt{DDT}(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}})$
- $\texttt{LDLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}}) = (-1)^{\Delta_{\mathrm{i}} \cdot \lambda_{\mathrm{i}}} \texttt{DLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}})$
- $\texttt{EDLCT}(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}}) = (-1)^{\lambda_{\mathrm{i}} \cdot \Delta_{\mathrm{i}} \oplus \lambda_{\mathrm{o}} \cdot \Delta_{\mathrm{o}}} \texttt{DDT}(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}})$
- $\texttt{LDLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}}) = \sum_{\Delta_{\mathrm{o}}} \texttt{EDLCT}(\Delta_{\mathrm{i}}, \Delta_{\mathrm{o}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}})$
- $\sum_{\Delta_{\mathrm{i}}} \texttt{LDLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{i}}, \lambda_{\mathrm{o}}) = \texttt{LAT}^2(\lambda_{\mathrm{i}}, \lambda_{\mathrm{o}})$

Properties of Generalized $\mathtt{DLCT}$ Tables - II

- $\mathtt{DDLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) = 2^{-n} \cdot \sum_{\Delta_m} \sum_{\lambda_m} \mathtt{UDLCT}\left(\Delta_{\mathrm{i}}, \Delta_m, \lambda_m\right) \cdot \mathtt{LDLCT}\left(\Delta_m, \lambda_m, \lambda_{\mathrm{o}}\right)$

$$\begin{aligned}
\mathtt{DDLCT}(\Delta_{\mathrm{i}}, \lambda_{\mathrm{o}}) &= \sum_{\Delta_m} \mathtt{DDT}(\Delta_{\mathrm{i}}, \Delta_m) \cdot \mathtt{DLCT}(\Delta_m, \lambda_{\mathrm{o}}) \\
&= 2^{-n} \sum_{\lambda_m} \mathtt{DLCT}(\Delta_{\mathrm{i}}, \lambda_m) \cdot \mathtt{LAT}^2(\lambda_m, \lambda_{\mathrm{o}}).
\end{aligned}$$

Results: Distinguishers for up to 17 Rounds of TWINE

- Comparing the data complexity of best boomerang and DL distinguishers

# Rounds	Boomerang [HNE22]	Differential-Linear	Gain
5	1	1	1
7	$2^{3.20}$	1	$2^{3.20}$
13	$2^{34.32}$	$2^{27.16}$	$2^{7.16}$
14	$2^{42.25}$	$2^{31.28}$	$2^{10.97}$
15	$2^{51.03}$	$2^{38.98}$	$2^{12.05}$
16	$2^{58.04}$	$2^{47.28}$	$2^{10.76}$
17	-	$2^{59.24}$	-

Results: Distinguishers for up to 17 Rounds of LBlock

- Comparing the data complexity of best boomerang and DL distinguishers

# Rounds	Boomerang [HNE22]	Differential-Linear	Gain
5	1	1	1
7	$2^{2.97}$	1	$2^{2.97}$
13	$2^{30.28}$	$2^{23.78}$	$2^{6.50}$
14	$2^{38.86}$	$2^{30.34}$	$2^{8.52}$
15	$2^{46.90}$	$2^{38.26}$	$2^{8.64}$
16	$2^{57.16}$	$2^{46.26}$	$2^{10.90}$
17	-	$2^{58.30}$	-

Results: Distinguishers for up to 8 Rounds of CLEFIA

- Comparing the data complexity of best boomerang and DL distinguishers

# Rounds	Boomerang [HNE22]	Differential-Linear	Gain
3	1	1	1
4	$2^{6.32}$	1	$2^{6.32}$
5	$2^{12.26}$	$2^{5.36}$	$2^{6.90}$
6	$2^{22.45}$	$2^{14.14}$	$2^{8.31}$
7	$2^{32.67}$	$2^{23.50}$	$2^{9.17}$
8	$2^{76.03}$	$2^{66.86}$	$2^{9.17}$

Results: Application to SERPENT

- 💻: Experimentally verified

Cipher	#R	$\mathbb{C}$	💻	Ref.
SERPENT	3	$\mathbf{2^{-0.68}}$	✓	This work
	4	$2^{-12.75}$		[DIK08]
	4	$\mathbf{2^{-5.54}}$	✓	This work
	5	$2^{-16.75}$		[DIK08]
	5	$\mathbf{2^{-11.10}}$	✓	This work
	8	$2^{-39.18}$		This work
	9	$2^{-56.50}$		[DIK08]
	9	$\mathbf{2^{-50.95}}$		This work

Results: Application to Simeck

- 💻: Experimentally verified

Cipher	#R	$\mathbb{C}$	💻	Ref.
	7	$\mathbf{1}$	✓	This work
Simeck-32	14	$2^{-16.63}$		[ZWH24]
	14	$\mathbf{2^{-13.92}}$	✓	This work

Cipher	#R	$\mathbb{C}$	💻	Ref.
	8	$\mathbf{1}$	✓	This work
	17	$2^{-22.37}$		[ZWH24]
	17	$\mathbf{2^{-13.89}}$	✓	This work
Simeck-48	18	$2^{-24.75}$		[ZWH24]
	18	$\mathbf{2^{-15.89}}$		This work
	19	$\mathbf{2^{-17.89}}$		This work
	20	$\mathbf{2^{-21.89}}$		This work

Cipher	#R	$\mathbb{C}$	💻	Ref.
	10	$\mathbf{1}$	✓	This work
	24	$2^{-38.13}$		[ZWH24]
	24	$\mathbf{2^{-25.14}}$		This work
Simeck-64	25	$2^{-41.04}$		[ZWH24]
	25	$\mathbf{2^{-27.14}}$		This work
	26	$\mathbf{2^{-30.35}}$		This work