Transistor: a TFHE-friendly Stream Cipher

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(joint work with Jules Baudrin, Sonia Belaïd, Nicolas Bon, Anne Canteaut, Gaëtan Leurent, Pascal Paillier, Léo Perrin, Matthieu Rivain, Yann Rotella, and Samuel Tap)

Overview

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1. Motivation
2. The TFHE Scheme
3. Our Design
4. Security Analysis

Plan

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Motivation

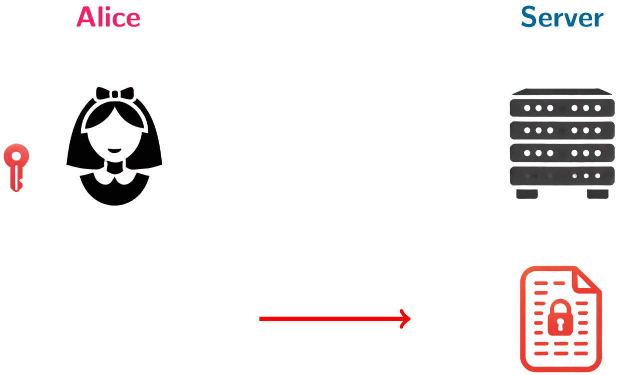
Alice wants to perform operations on her data but needs to **outsource** the processing.

Motivation

Alice   Server

Alice **encrypts** her data to protect its confidentiality.

Motivation



The server receives the encrypted data.

Motivation

The server needs to **decrypt** to process the data, which breaks confidentiality!

Fully Homomorphic Encryption (FHE)

FHE allows arbitrary computations to be performed directly on encrypted data **without needing to decrypt it**:

$$\text{Decrypt}(\text{Evaluate}(f, \text{Encrypt}(m))) = f(m)$$

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But:

- FHE operations are computationally intensive.
- Homomorphic ciphertexts are significantly larger than plaintexts, leading to increased communication costs.

Transciphering[1]

Alice wants to learn the application of f on her data m, by outsourcing the computation of $f(m)$ to the server.

Alice **Server**

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$$m \longrightarrow \boxed{E_K^S}$$

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$$m \longrightarrow \boxed{E^S_K}$$

$$K \longrightarrow \boxed{E^H}$$

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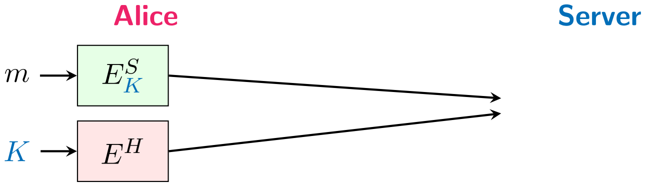

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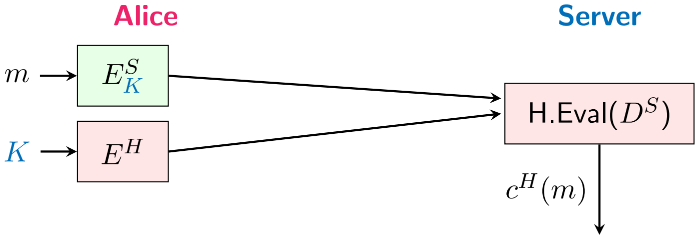

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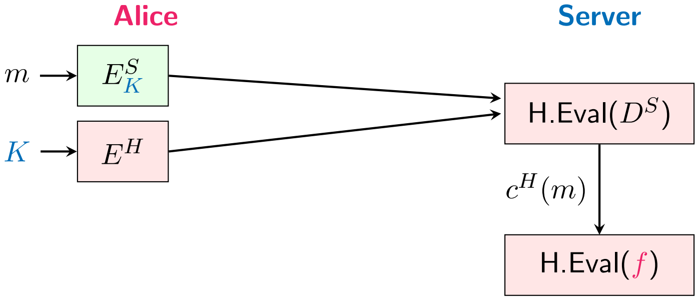

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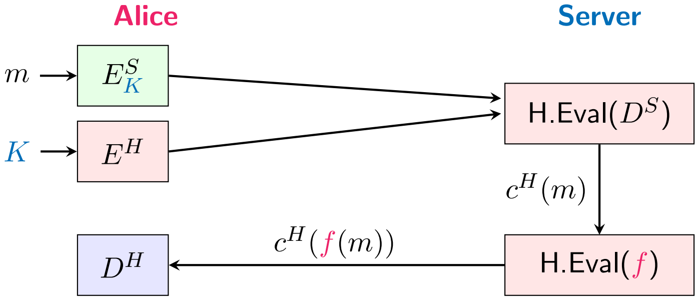

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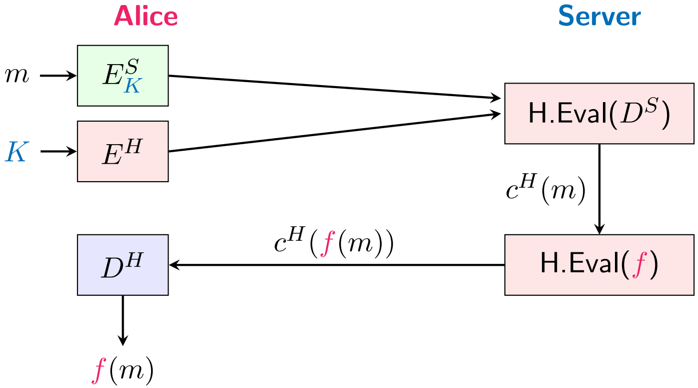

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Plan

TFHE (2016)

TFHE: Fast Fully Homomorphic Encryption over the Torus

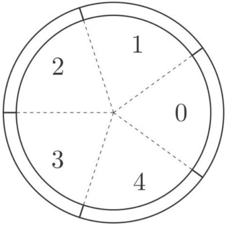
Ilaria Chillotti, Nicolas Gama, Mariya Georgieva, Malika Izabachène

- Based on the Learning With Errors (LWE) problem.
- Very fast for the FHE standards.
- Programmable bootstrapping: Evaluation of encrypted look-up tables (LUT) while resetting the noise level.
- **But**: Operations should be limited on small plaintexts (typically less than 6 bits).

TFHE: Description of the Scheme

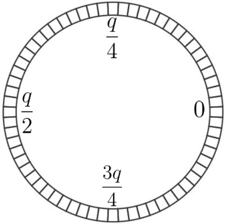
Discretized torus $\mathbb{T}_p = \{\frac{a}{p} \mid a \in \mathbb{Z}_p\}$.

Plaintext space $\mathbb{T}_p$



The size of $p \in \mathbb{N}$ is only a few bits.

Ciphertext space $\mathbb{T}_q$

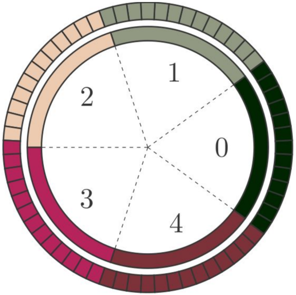


$q = 2^{32}$ or $q = 2^{64}$

Acknowledgment: Thanks to Nicolas Bon for the figures of TFHE.

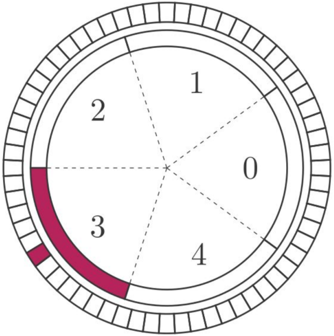
TFHE: Description of the Scheme

Natural embedding of $\mathbb{T}_p$ into $\mathbb{T}_q$: $m \mapsto \left\lfloor \frac{mq}{p} \right\rceil$.



Encryption Process

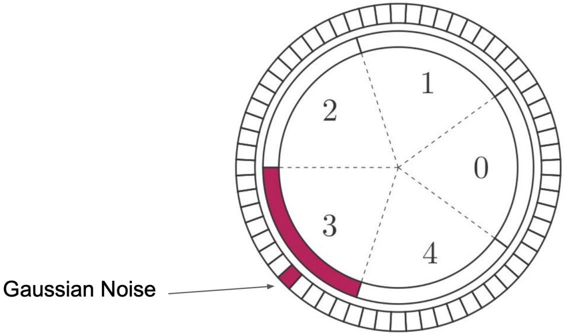
Encoding: $m \in \mathbb{T}_p \mapsto \widetilde{m} \in \mathbb{T}_q$



Encryption Process

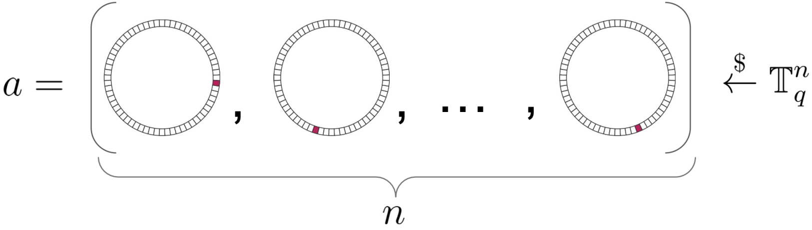
Encoding: $m \in \mathbb{T}_p \mapsto \widetilde{m} \in \mathbb{T}_q$

Adding noise: $\widetilde{m} + e$, with $e \xleftarrow{\$} \chi_\sigma$



Encryption Process

Mask $a = (a_1, \ldots, a_n) \in \mathbb{Z}_q^n$.

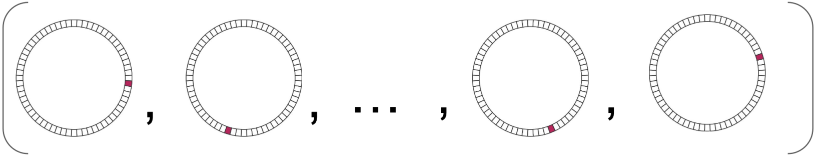


Secret key $\mathsf{sk} = (s_1, \ldots, s_n) \in \{0,1\}^n$.

Encryption Process

Ciphertext: $c = (a_1, \ldots, a_n, b)$

$c =$



where $b = \sum_{i=1}^{n} a_i \cdot s_i + \tilde{m} + e$.

Homomorphic Operations

Sum of ciphertexts

Let c_1 and c_2 be two ciphertexts encrypting m_1 and m_2 with noise levels σ_1^2 and σ_2^2, respectively. The **noise level of the ciphertext encrypting** $m_1 + m_2$ is $\sigma_1^2 + \sigma_2^2$.

Product with a cleartext

Let c be a ciphertext encrypting m with noise σ^2. Multiplying each coordinate of c by a constant $\lambda \in \mathbb{Z}$ produces a valid ciphertext c', which encrypts $m' = \lambda \cdot m$ with **noise** $\lambda^2 \cdot \sigma^2$.

Programmable Bootstrapping (PBS)

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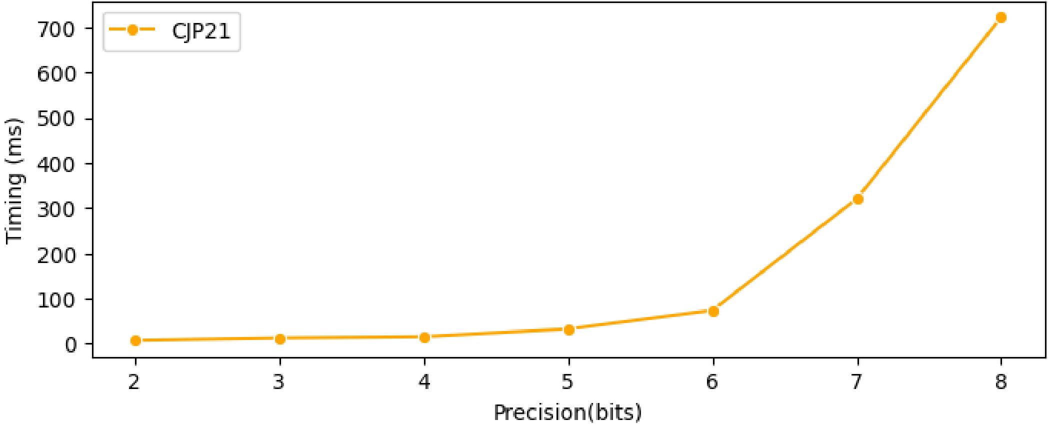
Bootstrapping

Generic technique introduced by Gentry that allows the noise of a ciphertext to be reset to a nominal level.

- In **TFHE**, bootstrapping is implemented in a programmable manner: while the noise is being reset, any arbitrary function can be evaluated on the ciphertext.

Timing of a PBS



Transciphering: State of the Art

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- LowMC (2016)
- Kreyvium (2016)
- FLIP (2016)
- FiLIP (2020)
- ~~Elisabeth~~ (2022)
- Elisabeth-b, Gabriel and Margrethe (2023) (patches of Elisabeth)
- FRAST (2024)

- Elisabeth (and its successors) as well as FRAST were designed specifically for **TFHE**.
- Trivium and Kreyvium provide good performance within the **TFHE** transciphering framework.

Plan

Design Choices

Plaintext space: $\mathbb{Z}_p$ with $p = 17$

- p is **odd** (avoid dealing with negacyclicity)
- p is close to 2^4 (convenient for **encoding nibbles**)
- p is **prime**: $\mathbb{Z}_p = \mathbb{F}_p$ has a field structure

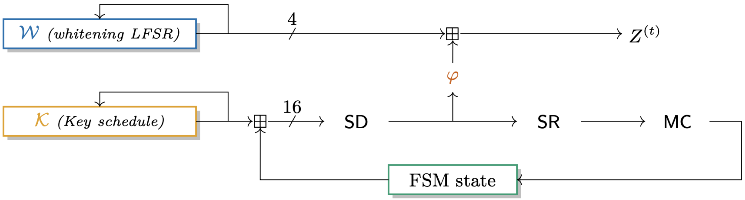
Non-linearity: S-box layer applying in parallel S: $\mathbb{Z}_{17} \to \mathbb{Z}_{17}$

- One PBS per S-box
- Minimize # PBS per element of the output stream

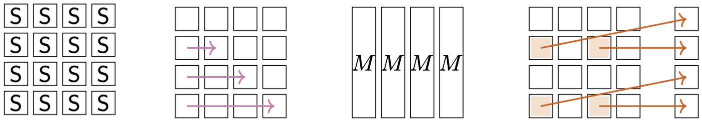
Transistor = Transciphering + Torus

- `Transistor` is a stream cipher that generates a keystream composed of elements from $\mathbb{F}_{17}$.
- It generates tuples of 4 digits at once.

A High Level View of `Transistor`



(a) General structure (rectangles correspond to registers).



(b) SD. (c) SR. (d) MC. (e) φ.

- $|\mathcal{K}| = 64,\ |\mathcal{W}| = 32.$

The S-box

Table representation

$$\mathsf{S} = [1, 12, 6, 11, 14, 3, 15, 5, 10, 9, 13, 16, 7, 8, 0, 2, 4]$$

Polynomial representation

$$\begin{aligned}\mathsf{S}(x) &= 1 + 4x^1 + 13x^2 + 7x^3 + 16x^4 + 15x^5 + 5x^7 + 5x^8 \\ &\quad + 11x^9 + 13x^{10} + 12x^{11} + 13x^{12} + 15x^{14} + x^{15}\ .\end{aligned}$$

Cryptographic properties

- Almost Perfect Nonlinear (APN) permutation:
  - $\mathsf{S}(x+a) = \mathsf{S}(x) + b$ has at most 2 solutions x for all $a \neq 0$ and all b.
- Maximum algebraic degree.

The Linear Layer (MC)

$$M = \begin{pmatrix} 2 & 1 & 1 & 1 \\ 1 & -1 & 1 & -2 \\ 1 & 1 & -2 & -1 \\ 1 & -2 & -1 & 1 \end{pmatrix}$$

Design criterion

M should be **MDS** and its ℓ_2-norm should be **as low as possible**.

The ℓ_2-norm of M is defined as:

$$\ell_2(M) = \max_{i=1,2,3,4} \sqrt{M(i,1)^2 + M(i,2)^2 + M(i,3)^2 + M(i,4)^2}.$$

The Silent LFSR

Homomorphic implementation of the LFSRs:

The naive approach: Maintain an encrypted state, and update it by computing a linear combination with the feedback coefficients.

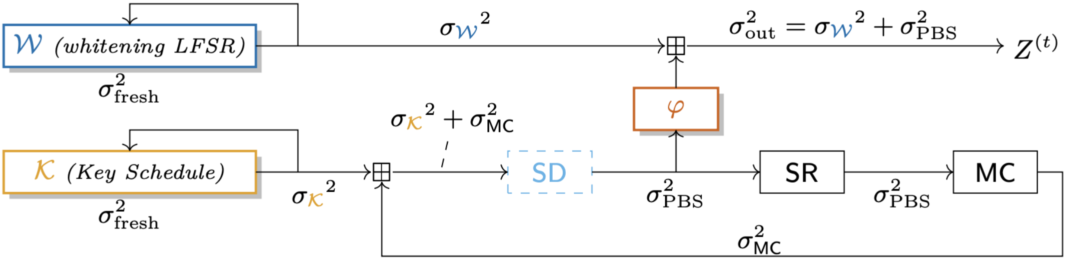
- Noise accumulates over time and needs periodic use of PBS operations to be refreshed.

The silent approach: Computing on the fly the coefficients of the linear combinations in clear, without updating an encrypted version of the internal state.

- The noise level remains stable over time.

Similar approach as in `FLIP` where a key state is queried **without being updated**.

Noise Evolution



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Security Analysis

Security Claim

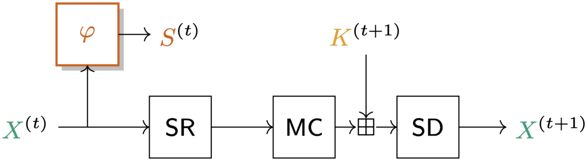
Transistor provides 128 bits of security, assuming no more than 2^{31} digits are generated with a single master key/IV pair.

We analyzed the security of the cipher against:

- Time-Memory-Data trade-off attacks
- Guess and determine attacks
- Fast correlation attacks
- Algebraic attacks

Guess and Determine Attack

The attacker links the FSM state $X^{(t)}$ to the filter output $S^{(t)}$ by guessing digits of $K^{(t)}$.



1. Observe $S^{(t)} = \varphi(X^{(t)}) = \mathsf{SD}\left(K^{(t)} + \left(\mathsf{MC} \circ \mathsf{SR}(X^{(t-1)})\right)\right)_{[4,6,12,13]}$.
2. Guess the 12 missing digits of $K^{(t)}$ to compute $X^{(t)}$.

Complexity: $p^{\frac{3}{4}|\mathcal{K}|+|\mathcal{W}|}$ ($p = 17$, $|\mathcal{K}| = 64$ and $|\mathcal{W}| = 32$).

(Fast) Correlation Attacks

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Objective

Recover information about the initial state from the knowledge of the keystream.

Question: What is the smallest length of output sequence $(S^{(t)})$ that can provide information on the key-LFSR?

Theorem

The output of

$$F^{(3)} : (X^{(t)}, K^{(t+1)}, K^{(t+2)}) \mapsto (S^{(t)}, S^{(t+1)}, S^{(t+2)})$$

is statistically independent from (i.e., not correlated to) the key sequence.

Four consecutive outputs

Question: What is the data complexity to recover the internal state from the knowledge of at least four consecutive outputs $S^{(t)}, S^{(t+1)}, S^{(t+2)}, S^{(t+3)}$?

Proposition (Xiao-Massey lemma over $\mathbb{F}_p$)

If the output of

$$F^{(n)} : (X^{(t)}, K^{(t+1)}, \ldots, K^{(t+n-1)}) \mapsto (S^{(t)}, S^{(t+1)}, \ldots, S^{(t+n-1)})$$

is correlated to its key-input, then there exists a biased linear relation between the key-inputs and the outputs of $F^{(n)}$.

Data complexity of fast correlation attacks

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The **data complexity** of the best correlation attack based on a linear approximation

$$\sum_{i=1}^{n-1} \alpha_i \cdot K^{(t+i)} + \sum_{i=0}^{n-1} \beta_i \cdot S^{(t+i)}, \forall t \geq 0$$

is the inverse of

$$\Delta = \frac{p}{64 \ln p} \left(\frac{\mathcal{L}(\mathsf{S})}{p} \right)^{2w_n}$$

where

- $\mathcal{L}(\mathsf{S})$: maximal modulus of the Fourier coefficients of S
- $w_n = \sum_{i=1}^{n-1} wt(\alpha_i)$: # active S-boxes in the linear trail.

With MILP-based search

$w_4 \geq 13$, $w_5 \geq 20$, $w_6 \geq 25$, and $w_n \geq 26$ for $n \geq 7$.

Benchmarks

Cipher	p_{err}	Setup	Latency	Throughput
`FRAST`	2^{-80}	25 s (8 threads)	6.2 s	20.66 bits/s
`Transistor`	2^{-128}	No	251 ms	65.10 bits/s

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Thanks for your attention!