

Sample Question

IAI – Ph.D. in Engineering Admission

Each question carries 2 marks. Provide brief explanations for your answers.

Negative marking is applicable.

Answer Group A (mandatory) and either Group B or C.

Group A

1. Which of the following statements are True?

- (a) Let A be a 3×3 real matrix satisfying $A^3 = \text{Id}$. Then A is diagonalisable over \mathbb{R} .
- (b) Let A be a 3×3 real matrix satisfying $A^3 = \text{Id}$. Then A is diagonalisable over \mathbb{C} .
- (c) Let A be a 3×3 real matrix satisfying $AA^t = A^tA = \text{Id}$ and $\det(A) = 1$. Then 1 is an eigen value of A .

2. Which of the following statements are false.

- (a) Let A be $n \times n$ real matrix such that $AB = BA$ for all invertible matrix B . Then $A = \lambda \text{Id}$ for some $\lambda \in \mathbb{R}$.
- (b) Let A be $n \times n$ real matrix such that $AB = BA$ for all diagonal matrix B . Then $A = \lambda \text{Id}$ for some $\lambda \in \mathbb{R}$.
- (c) Let A be $n \times n$ real matrix such that $AB = BA$ for all upper triangular matrix B . Then $A = \lambda \text{Id}$ for some $\lambda \in \mathbb{R}$.

3. Let

$$A = \begin{bmatrix} 1 & 0 & 2 & 3 & 5 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 2 & 0 \end{bmatrix}$$

Then which of the following statements are true

- (a) The first two rows are linearly independent.
- (b) The last two columns are linearly independent.
- (c) The dimension of the vector space generated by last 4 rows is 3.

4. Consider the differential equation

$$y''' + y'' - 33y' + 63y = 0.$$

For any real numbers c_1, c_2 and c_3 , which of the following is a solution of the above differential equation

- (a) $c_1e^{3x} + c_2xe^{3x} + c_3e^{-7x}$.
- (b) $c_1e^{-3x} + c_2xe^{-3x} + c_3e^{7x}$.
- (c) $c_1e^{3x} + c_2e^{7x}$.
- (d) $c_1e^{3x} + c_2e^{-7x} + c_3xe^{-7x}$.

5. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be the function defined by $f(x, y) = x^2 - \sqrt[3]{2}xy^2$. Then the point $(0, 0)$ is one of the following.

- (a) A relative maximum.

- (b) A relative minimum.
(c) A saddle point.
(d) None of the above.
6. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the function defined by $f(x, y) = (x^3 + 2y, x - y)$. Then the line integral of f from $(-1, 1)$ to $(1, 1)$ along the parabola $y = x^2$ is
- (a) $\frac{4}{3}$.
(b) $\frac{6}{3}$.
(c) $\frac{8}{3}$.
(d) $\frac{2}{3}$.
7. A laboratory blood test correctly identifies a certain disease 90% of the time when the disease is actually present. However, the test also produces a "false positive" result in 1% of cases where the person is healthy (meaning that if a healthy individual is tested, there is a 0.01 probability that the test will incorrectly indicate they have the disease). Given that 1% of the population actually has the disease, what is the probability that a person has the disease if their test result comes back positive? (correct upto second place of decimal)
- (a) 0.08
(b) 0.48
(c) 0.01
(d) 0.99

8. The PDF of a gamma distribution is given by

$$g(x, \alpha, \lambda) = \frac{1}{\lambda^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-\frac{x}{\lambda}} \quad x > 0, \alpha > 0, \lambda > 0 \quad (1)$$

$$0 \quad \text{otherwise} \quad (2)$$

Find its expected value / mean.

The definition of Γ is $\Gamma = \int_a^b x^{\alpha-1} e^{-x} dx$. You can assume that the relationship $\Gamma[\alpha + 1] = \alpha\Gamma[\alpha]$ holds.

- (a) $\lambda\alpha$
(b) $\frac{\lambda}{\alpha}$
(c) $\frac{\alpha}{\lambda}$
(d) $\lambda + \alpha$
9. The probability mass function $f(x)$ of a discrete random variable is 0 except at the points 0, 1, 2; at these points its values are:
 $f(0) = 2c - 1$; $f(1) = 3c - 2c^2$; $f(2) = 2c - 1$.
- (a) 0
(b) $\frac{1}{2}$
(c) 3
(d) 1
10. The ratios of prices in 2016 to those in 2006 for food, cosmetics, travel and entertainment are 0.92, 1.25, 1.75 and 0.85 respectively. What is the average price index? (round-off to second place of decimal)
- (a) 1.19
(b) 1.14
(c) 1.10
(d) none of the above

Group B

1. Consider a continuous-time signal $x(t)$ with its Fourier transform given by:

$$X(j\omega) = \frac{4j\omega}{j\omega + 2} + \frac{5}{j\omega + 4} + \frac{10(j\omega - 5)}{(j\omega)^2 + 100} + \delta(\omega - 12\pi)$$

The signal $x(t)$ is sampled at a rate of 24 Hz. Which of the following statements is correct? Show your justification for the chosen option.

- (a) The sampled signal will experience aliasing due to $\delta(\omega - 12\pi)$ component because its frequency exceeds the Nyquist rate.
 - (b) The sampled signal will not experience aliasing as all components are below the Nyquist rate.
 - (c) The sampled signal will experience aliasing in the $\frac{5}{j\omega + 4}$ component because it allows frequencies that could fold back into the spectrum.
 - (d) The sampled signal will experience aliasing in the $\frac{10(j\omega - 5)}{(j\omega)^2 + 100}$ component because it allows frequencies that might overlap with the Nyquist frequency.
2. Consider a discrete-time signal $x[n]$ with the z-transform given by:

$$X(z) = \frac{(z^2 - 1.5z + 0.5)(z^2 - 0.7z + 0.1)}{(z - 0.6)(z - 1.2)(z - 2.0)(z + 0.4)}$$

Assuming $x[n]$ is a causal signal, which of the following statements best describes the system's stability and filtering characteristics given that the Region of Convergence (ROC) is $|z| > 2.0$? Show your justification for the chosen option.

- (a) The ROC is $|z| > 2.0$. The system is BIBO stable and exhibits a band-stop filter characteristic.
 - (b) The ROC is $|z| > 2.0$. The system is BIBO stable and exhibits a low-pass filter characteristic.
 - (c) The ROC is $|z| > 2.0$. The system is BIBO unstable and exhibits a high-pass filter characteristic.
 - (d) The ROC is $|z| > 2.0$. The system is BIBO stable and exhibits a notch filter characteristic.
3. For a Linear Time-Invariant (LTI) system with an impulse response: $h(t) = e^{-(t-2)}u(t)$, where $u(t)$ is the unit step function, which of the following statements is true? Show your justification for the chosen option.
- (a) The system is neither causal nor stable and the group delay at $\omega = 1$ rad/s is 1 second.
 - (b) The system is causal and stable and the group delay at $\omega = 1$ rad/s is 0.5 seconds.
 - (c) The system is non-causal but stable and the group delay at $\omega = 1$ rad/s is 0.5 seconds.
 - (d) The system is causal and unstable and the group delay at $\omega = 1$ rad/s is 1 second.

4. Let $y[n]$ represent the out of an LTI system with causal input sequence $g[n]$ and response $h[n]$, where $h[n] = \left(\frac{1}{2}\right)^n u[n]$. Given that $y[0] = 1$ and $y[1] = \frac{1}{2}$, determine the value of $g[1]$.
- (a) 0
 - (b) $\frac{1}{2}$
 - (c) 1
 - (d) $\frac{3}{2}$

5. Consider a linear time-invariant continuous-time system with an impulse response given by

$$h(t) = e^{at}u(t) + e^{bt}u(-t),$$

where $u(t)$ represents the unit step function, and a and b are real constants. Determine the conditions under which the system is stable:

- (a) $a > 0$ and $b > 0$
- (b) $a < 0$ and $b < 0$
- (c) $a > 0$ and $b < 0$
- (d) $a < 0$ and $b > 0$

6. The Fourier transform $X(j\omega)$ of the signal

$$x(t) = \frac{t}{(1+t^2)^2}$$

is given by ----.

- (a) $\frac{\pi}{2j}\omega e^{-|\omega|}$
- (b) $\frac{\pi}{2}\omega e^{-|\omega|}$
- (c) $\frac{\pi}{2j}e^{-|\omega|}$
- (d) $\frac{\pi}{2}e^{-|\omega|}$

7. For a sequence $x[n]$ with $0 \leq n \leq N - 1$, the Discrete Fourier Transform (DFT) X is given by

$$X[k] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}nk}, \quad 0 \leq k \leq N - 1.$$

For $N = 4$, which of the following sequences satisfies $\text{DFT}(\text{DFT}(x)) = x$?

- (a) $x = [1, 2, 3, 4]$
- (b) $x = [1, 2, 3, 2]$
- (c) $x = [1, 3, 2, 2]$
- (d) $x = [1, 2, 2, 3]$

8. You have a speech signal with a sampling frequency of 8 kHz, a duration of 1 minute, and each sample is encoded using 16 bits. What is the total amount of data (in kilobytes) required to store this speech signal?

- (a) 480 kB
- (b) 960 kB
- (c) 1,600 kB
- (d) 8,000 kB

9. Consider a binary communication system where the probability of transmitting a '1' is $p(1) = 0.7$ and the probability of transmitting a '0' is $p(0) = 0.3$. What is the entropy of this system in bits?

- (a) 0.88 bits
- (b) 0.92 bits
- (c) 0.97 bits
- (d) 1.00 bits

10. A digital communication system transmits data at a bit rate of 1 Mbps over a channel with a bandwidth of 200 kHz. According to the Nyquist criterion, what is the minimum signal-to-noise ratio (SNR) in dB required to achieve this bit rate without any error?

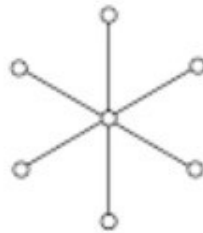
- (a) 3 dB
- (b) 6 dB
- (c) 9 dB
- (d) 12 dB

Group C

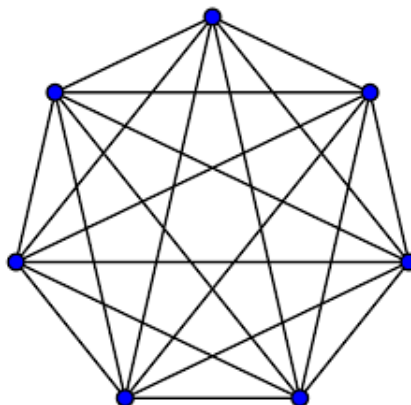
1. What are the allowed values for height of an AVL tree?
 - (a) 0,1
 - (b) 2,0,-2
 - (c) 0,1,2
 - (d) -1,0,1
2. A heuristic is a way

 - (a) To discover something or an idea embedded in a program
 - (b) To compare two nodes in a search tree to see if one is better than other
 - (c) To search and measure how far a node in a search tree seems to be from goal
 - (d) All the Above

3. Which of the following graphs is not a regular graph?
 - (a) Isolated Graph
 - (b) Cycle
 - (c) Complete Graph
 - (d) Wheel
4. What is the chromatic number of the tree?



- (a) 7
 - (b) 2
 - (c) 6
 - (d) 1
5. For the given graph (G), which of the following statements is true?



- (a) The covering number of the graph is 1
 - (b) G is not a connected graph
 - (c) Independence number of the graph is 1
 - (d) Both (a) and (c)
6. Which of the following statement is true?
- (a) Two simple graphs G and H are isomorphic if and only if complement of G is isomorphic to complement of H .
 - (b) Two simple graphs G and H are isomorphic if and only if number of vertices in H and number of vertices in G are same.
 - (c) Two simple graphs G and H are isomorphic if and only if number of edges in G and number of edges in H are same.
 - (d) None of these
7. If a set is uncountably infinite, which of the following must be true?
- (a) It is a subset of the set of all integers
 - (b) It can be put into a one-to-one correspondence with the set of all natural numbers
 - (c) It has the same cardinality as the set of all real numbers
 - (d) It has a finite number of elements
8. Which of the following statement is a proposition?
- (a) Get me a glass of milkshake
 - (b) God bless you!
 - (c) What is the time now?
 - (d) The only odd prime number is 2
9. Which of the following statements are TRUE about a minimal spanning tree?
- (a) Each branch has a weight.
 - (b) Contains no cycles.
 - (c) All vertices are connected.
 - (d) The path results in the minimum total weight.
10. Type 1 Grammar of Chomsky hierarchy is called as
- (a) Unrestricted Grammar
 - (b) Regular Grammar
 - (c) Context Free Grammar
 - (d) Context Sensitive Grammar