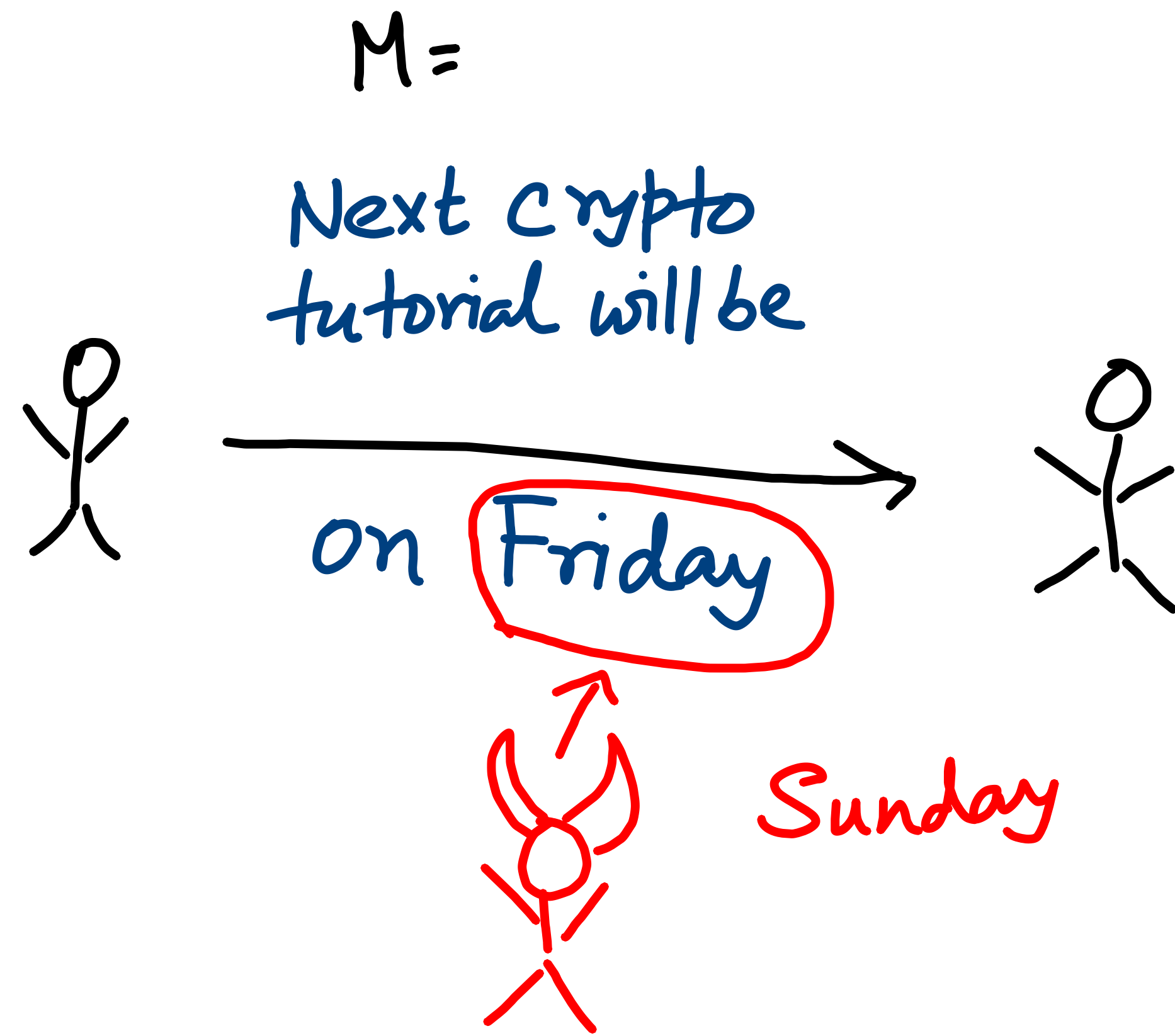
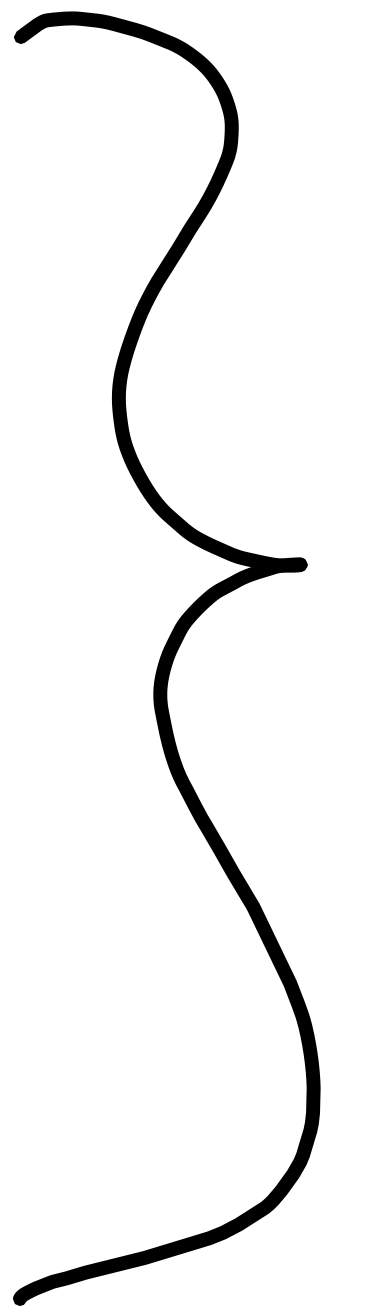


Message Authentication Code

(MAC)



- No privacy is required
- Integrity / Authenticity is required.



$$\text{MAC} = (\text{KG}, \text{TG}, \text{Vrfy})$$

$$k \leftarrow \text{KG}(1^n)$$

$$t \leftarrow \text{TG}(k, m)$$

$$\begin{array}{l} \text{T} / \text{I} \\ \text{=} / \text{=} \\ \text{1} / \text{0} \end{array} \leftarrow \text{Vrfy}(k, (m, t))$$

TG \rightarrow Tag Generation

Verify algo

Vrfy($k, (m, t)$)

{

$t' \leftarrow \text{TG}(k, m)$

if ($t == t'$)

else return T/1

return I/0

}

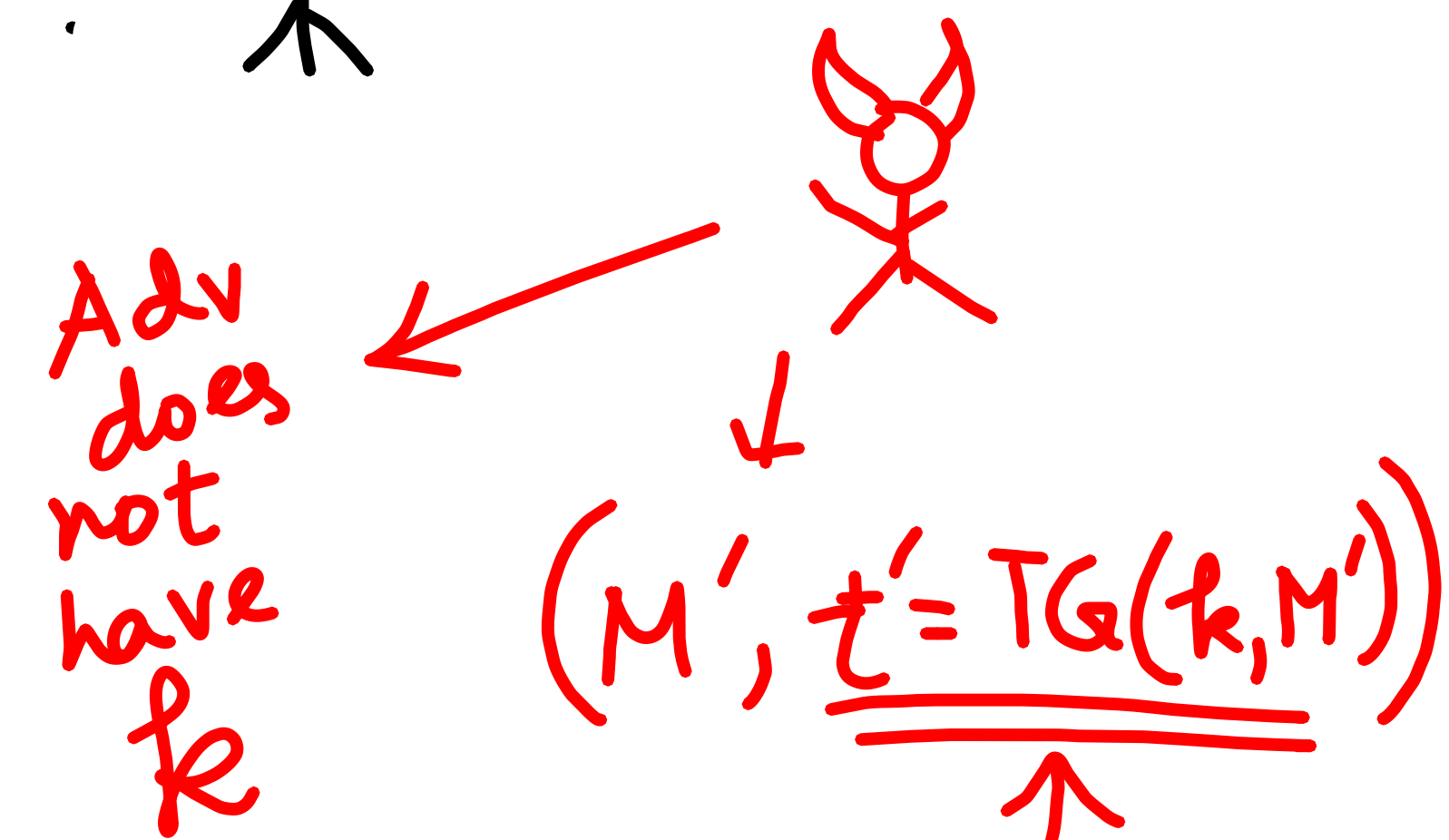
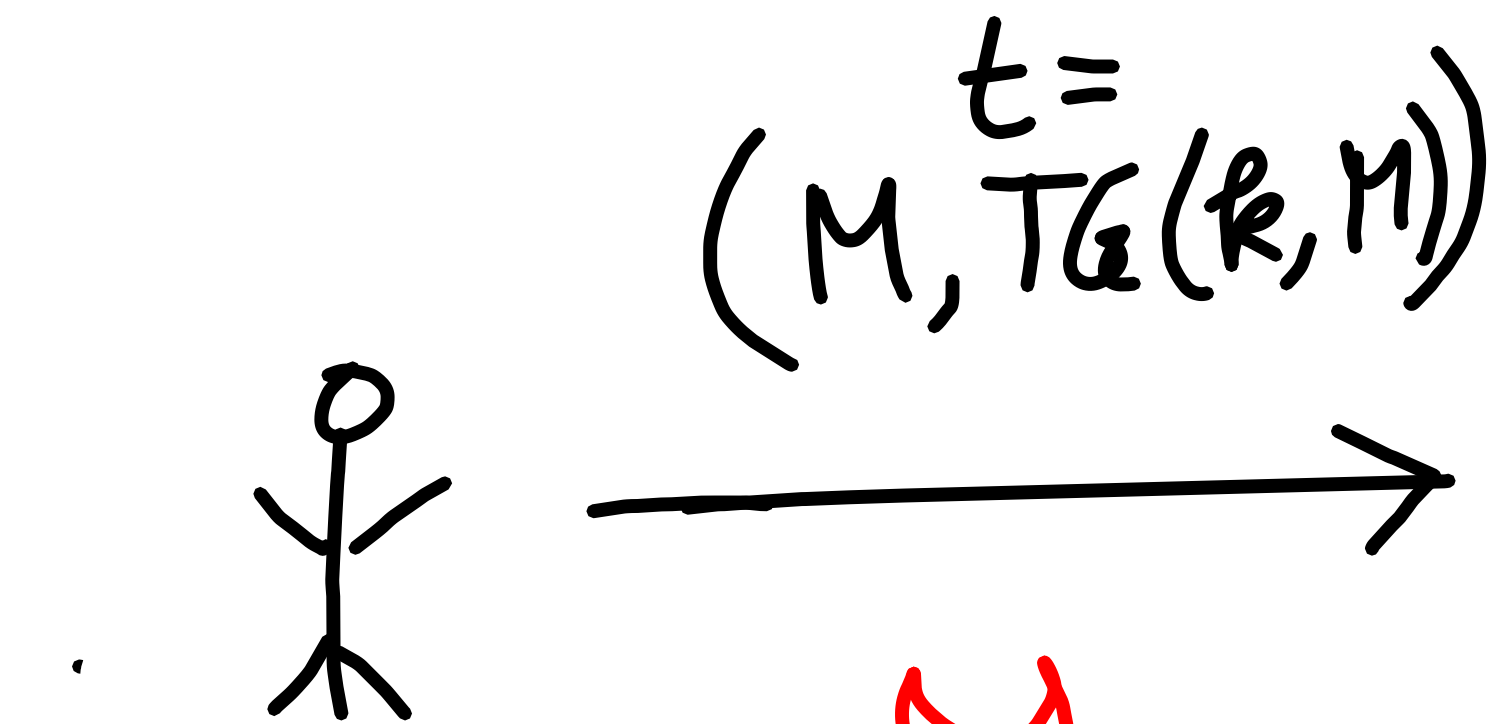
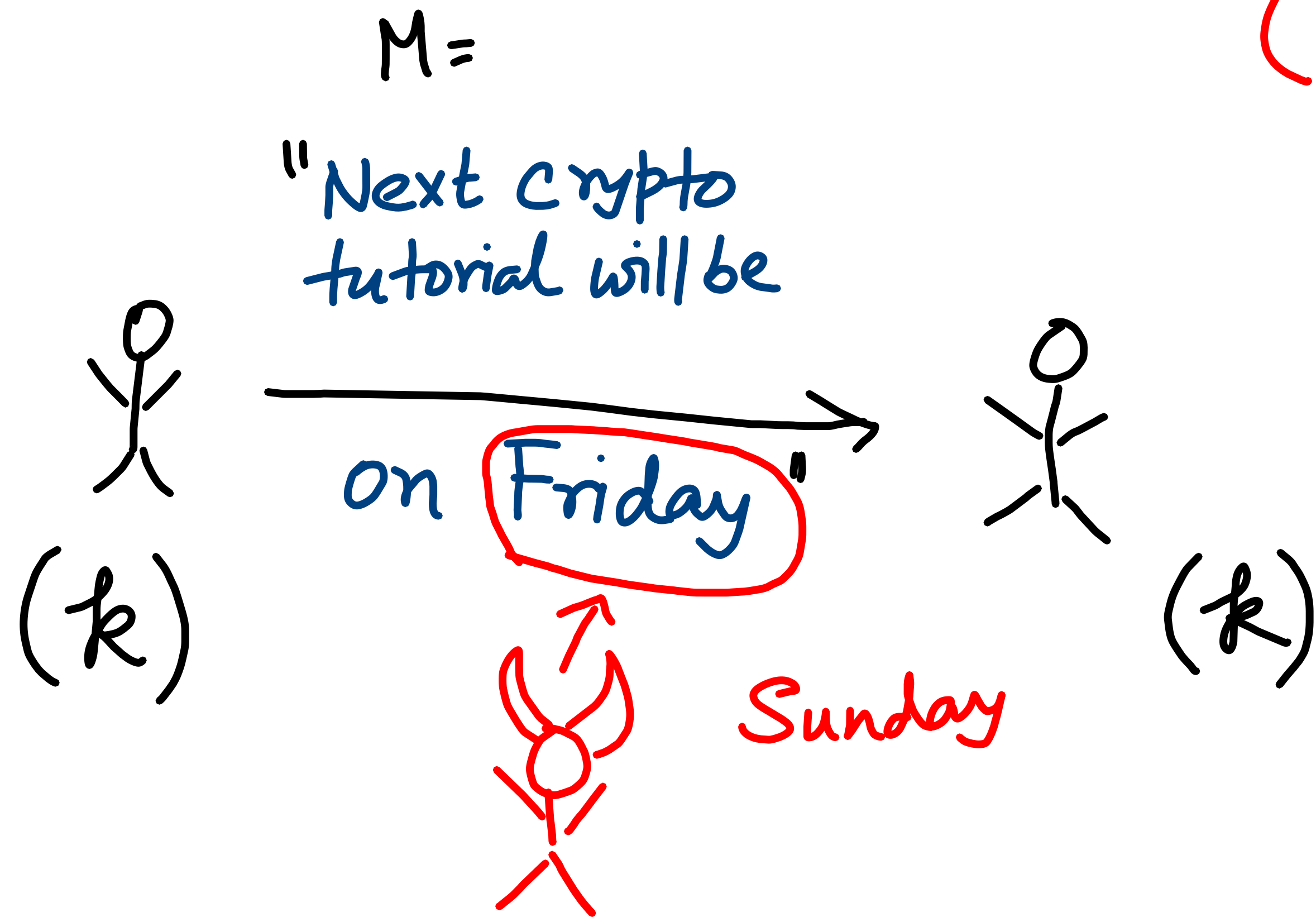
I \rightarrow abort
or
unsuccessful

T \rightarrow top or
successful

Message Authentication Code

(MAC)

$$\text{MAC} = (\text{KG}, \text{TG}, \text{VR}, \text{FY})$$



$$\text{If } (\text{VR}, \text{FY}(K, (m, t)) = T/1) \text{ then read } M$$

else

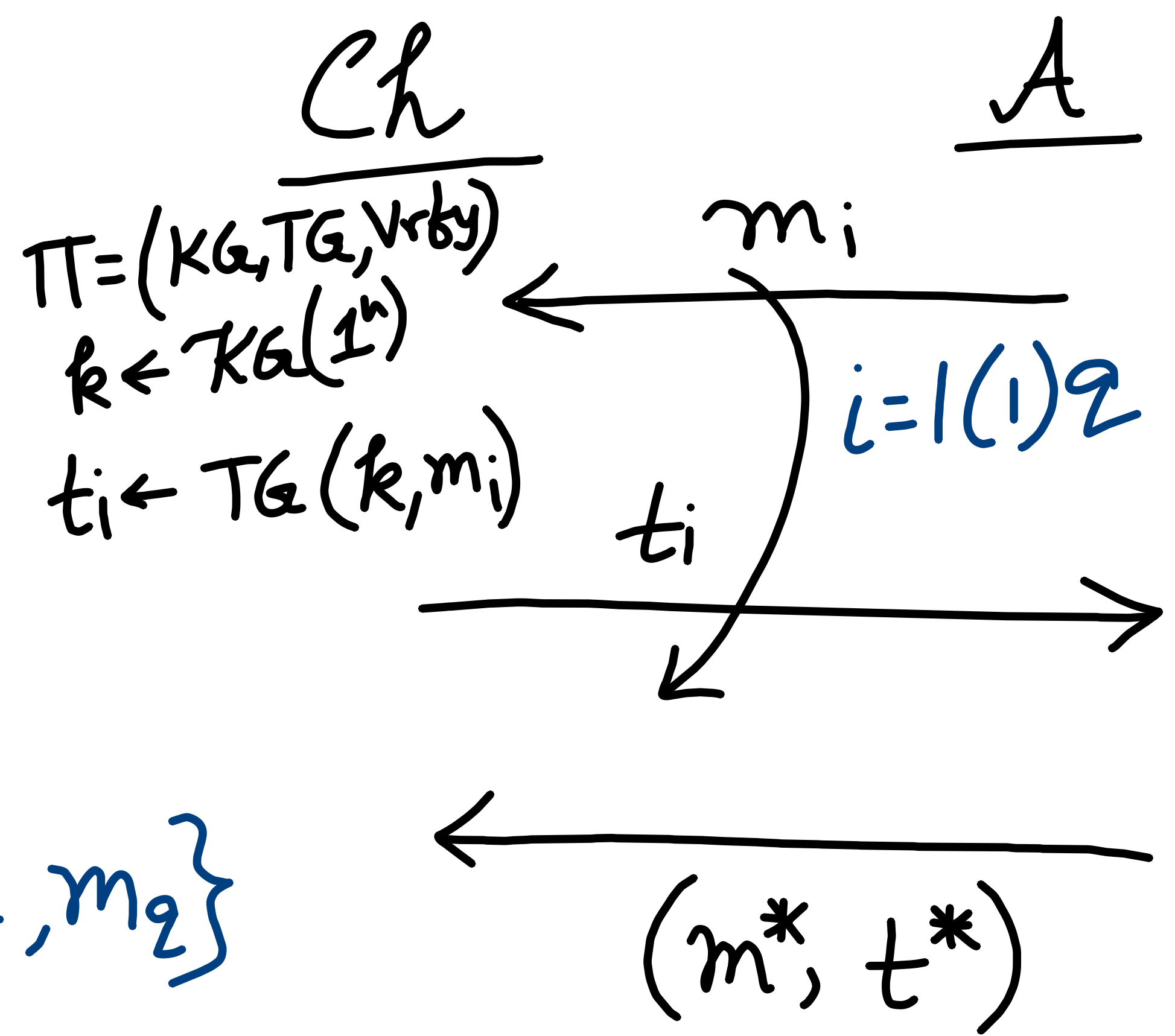
Hard to calculate for the adversary

There are some modification

MAC - Security

EUF → existential unforgeability
 CMA → chosen message attack

Existential Unforgeability



A forges if

- (fresh) → (i) $m^* \notin \{m_1, \dots, m_q\}$
- (valid) → (ii) $Vrfy(k, (m^*, t^*)) = T/1$

$$MAC_{\Pi}(A) = 1$$

if A forges

$\Pi \rightarrow$ EUF-CMA Secure

$$\Pr[MAC_{\Pi}(A) = 1]$$

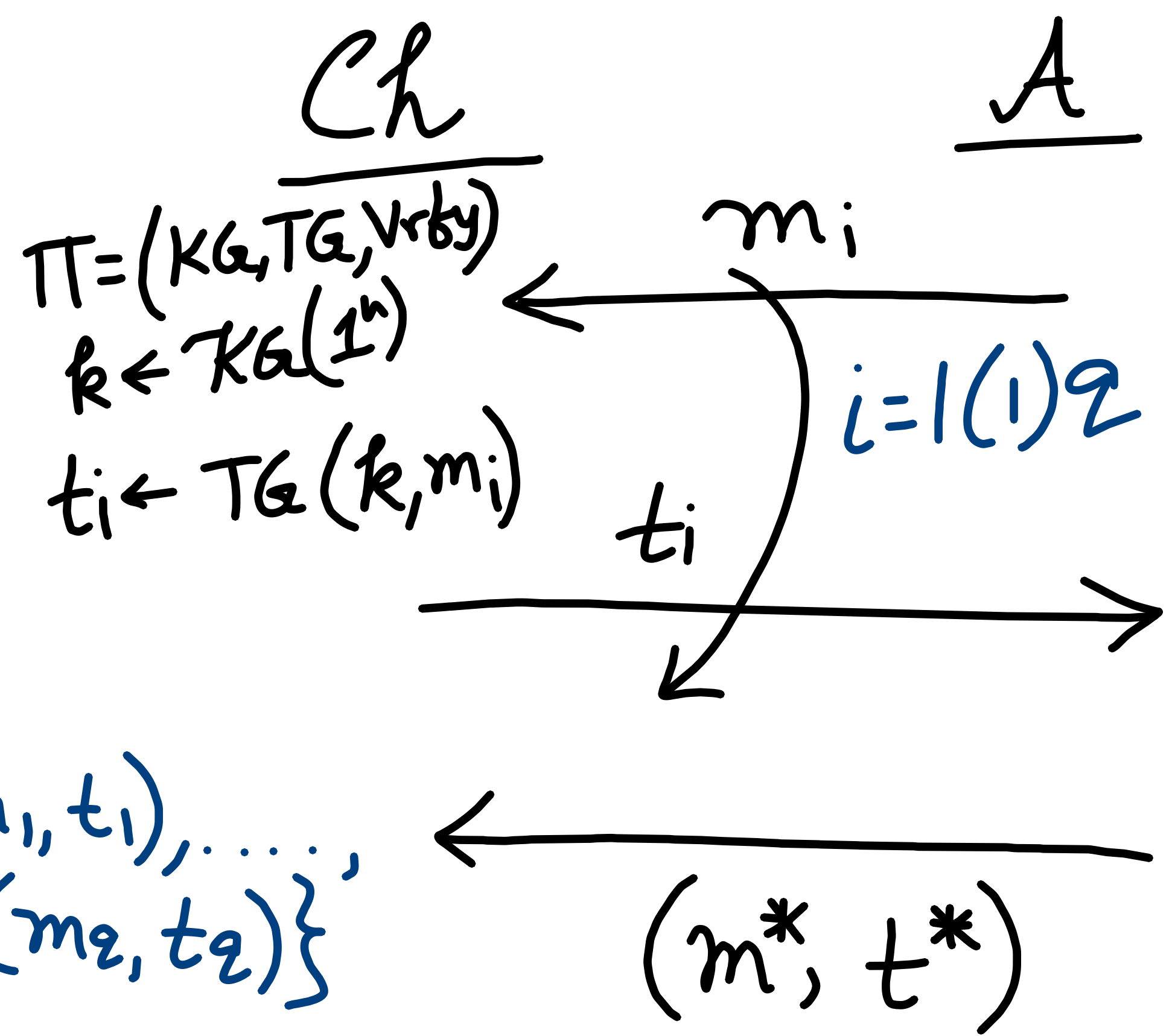
$$\leq \text{negl}(n)$$

\forall PPT A.

MAC - Security

SUF \rightarrow Strong unforgeability
 CMA \rightarrow Chosen message attack

Strong Unforgeability



A forges if

(fresh) \rightarrow (i) $(m^*, t^*) \notin \{(m_1, t_1), \dots, (m_q, t_q)\}$

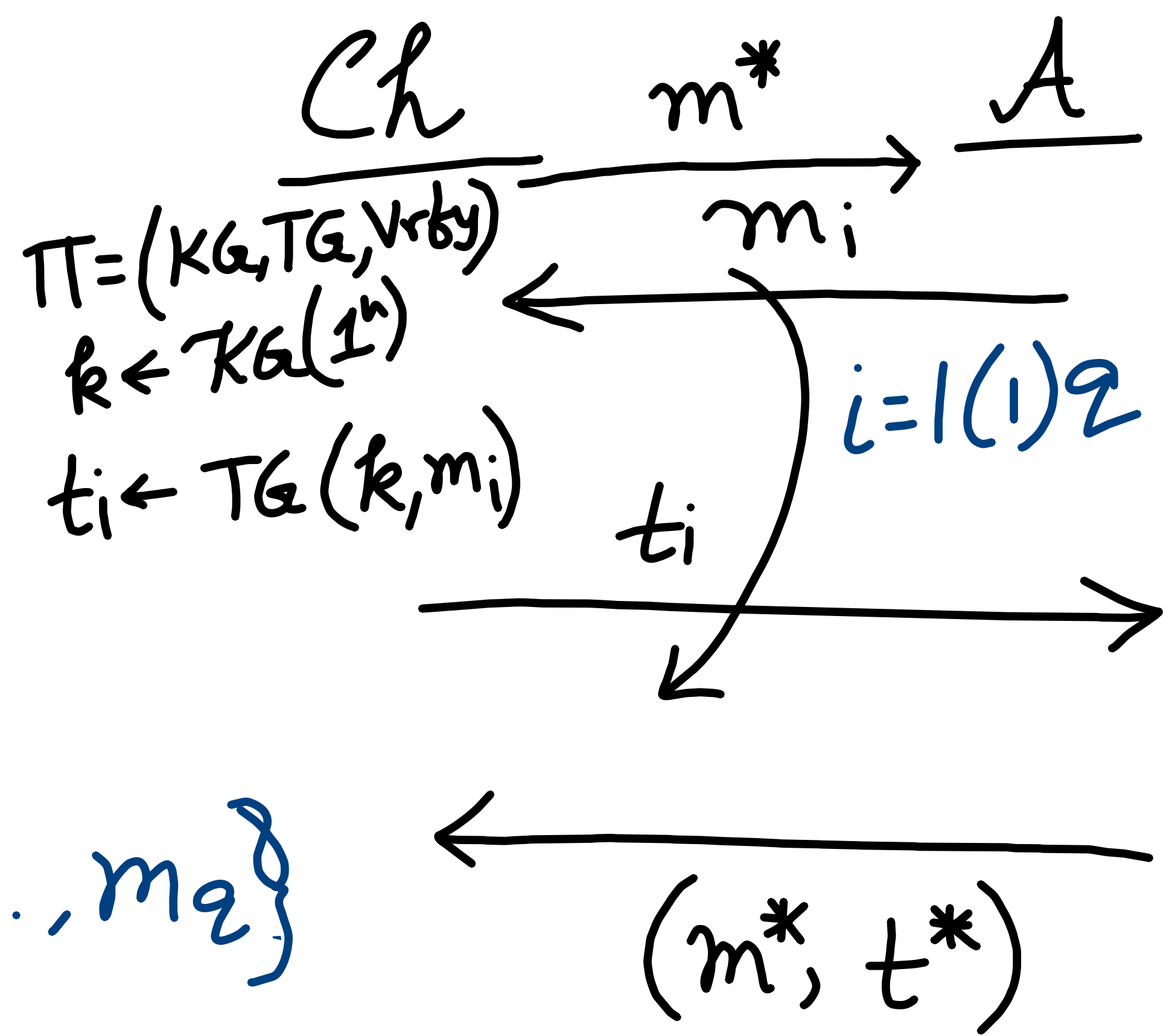
(valid) \rightarrow (ii) $Vrfy(k, (m^*, t^*)) = T/1$

$MAC_{\Pi}(A) = 1$
 if A forges
 $\Pi \rightarrow$ SUF-CMA Secure
 if $Pr[MAC_{\Pi}(A) = 1] \leq \text{negl}(n)$
 \forall PPT A.

MAC - Security

UUF \rightarrow Universal unforgeability
 CMA \rightarrow Chosen message attack

Universal
Unforgeability



A forges if

(fresh) \rightarrow (i) $m^* \notin \{m_1, \dots, m_q\}$

(valid) \rightarrow (ii) $Vrfy(k, (m^*, t^*)) = T/1$

$MAC_{\Pi}(A) = 1$
 if A forges
 $\Pi \rightarrow$ UUF-CMA Secure
 if $Pr[MAC_{\Pi}(A) = 1] \leq \text{negl}(n)$
 \forall PPT A.

MACs for Fixed-length Message

$$\Pi = (\text{KG}, \text{TG}, \text{Vrfy})$$

$$m \in \{0,1\}^n$$

$$\left\{ \begin{array}{l} \text{KG} \rightarrow k \leftarrow \{0,1\}^k \\ \text{TG} \rightarrow t \leftarrow F_k(m) \\ \text{Vrfy} \rightarrow \text{Vrfy}(k, (m, t)) \end{array} \right. \quad \Bigg| \quad F \rightarrow \text{PRF}$$

Π^m

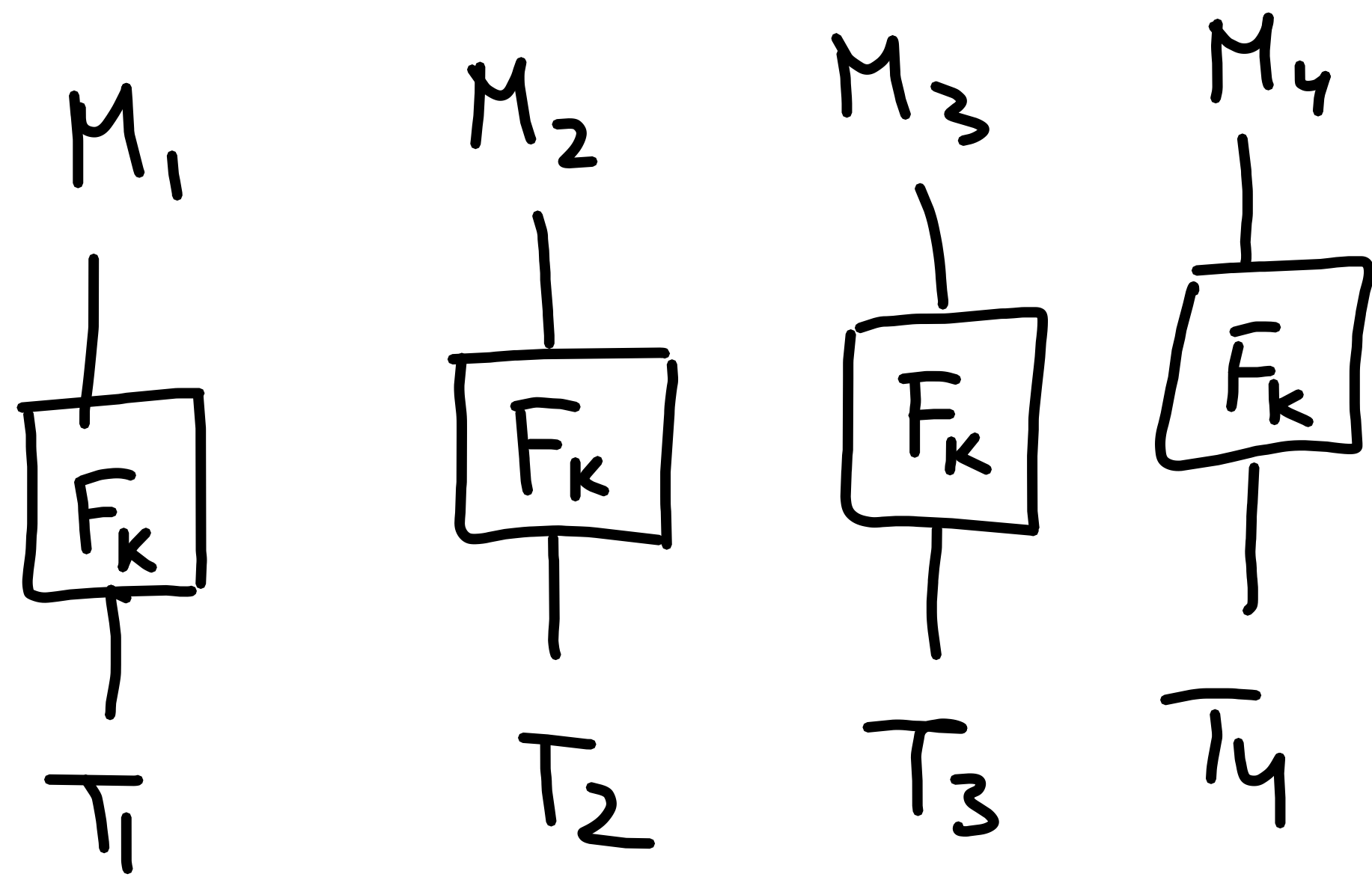
If F is PRF then

Π is EUF-CMA Secure

Try Reduction

$t'' \leftarrow F_k(m')$
if $(t' == t'')$
else 0 then $T/2$

MACs for Variable length Messages



$$TG(k, M_1 || M_2 || M_3 || M_4) = (T_1 || T_2 || T_3 || T_4)$$

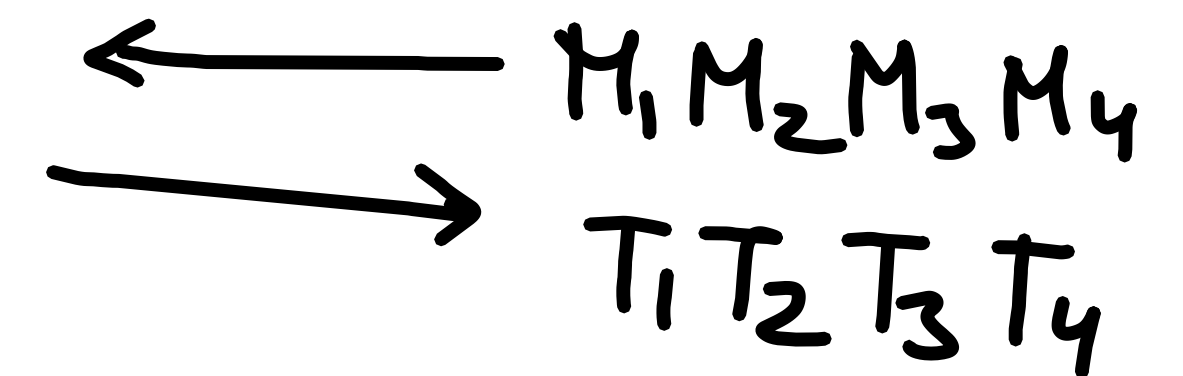
Is it EUF-CMA Secure?

No

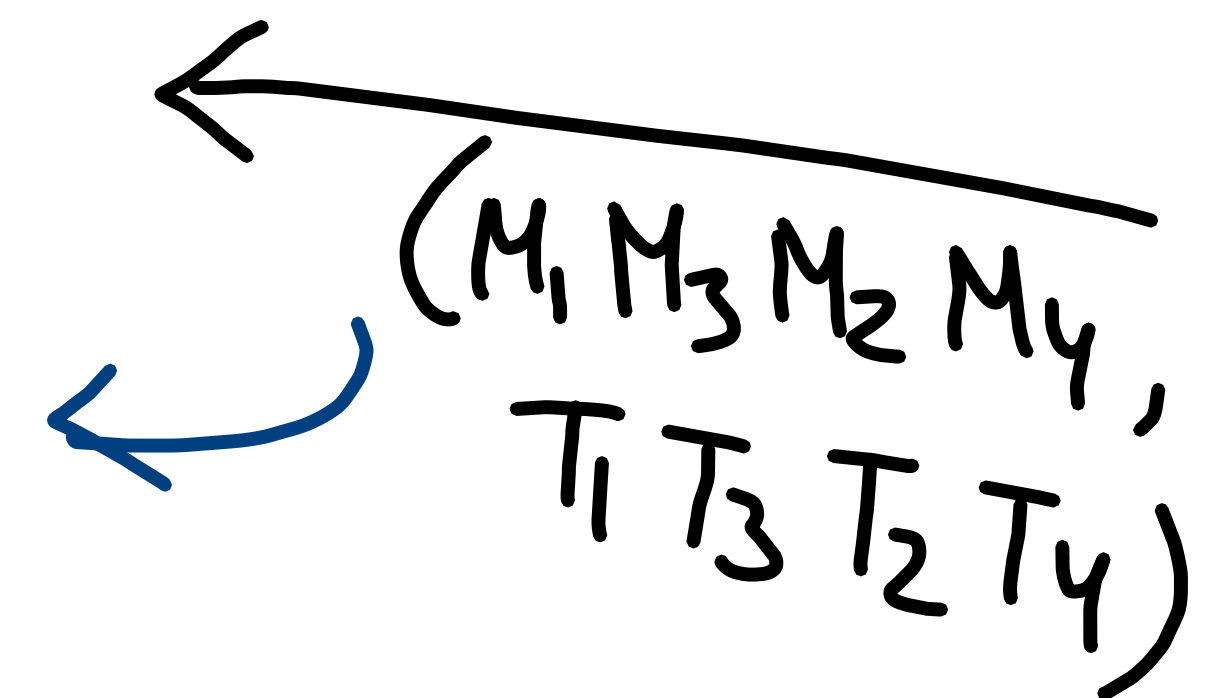
(Re-arrange the blocks)

CR

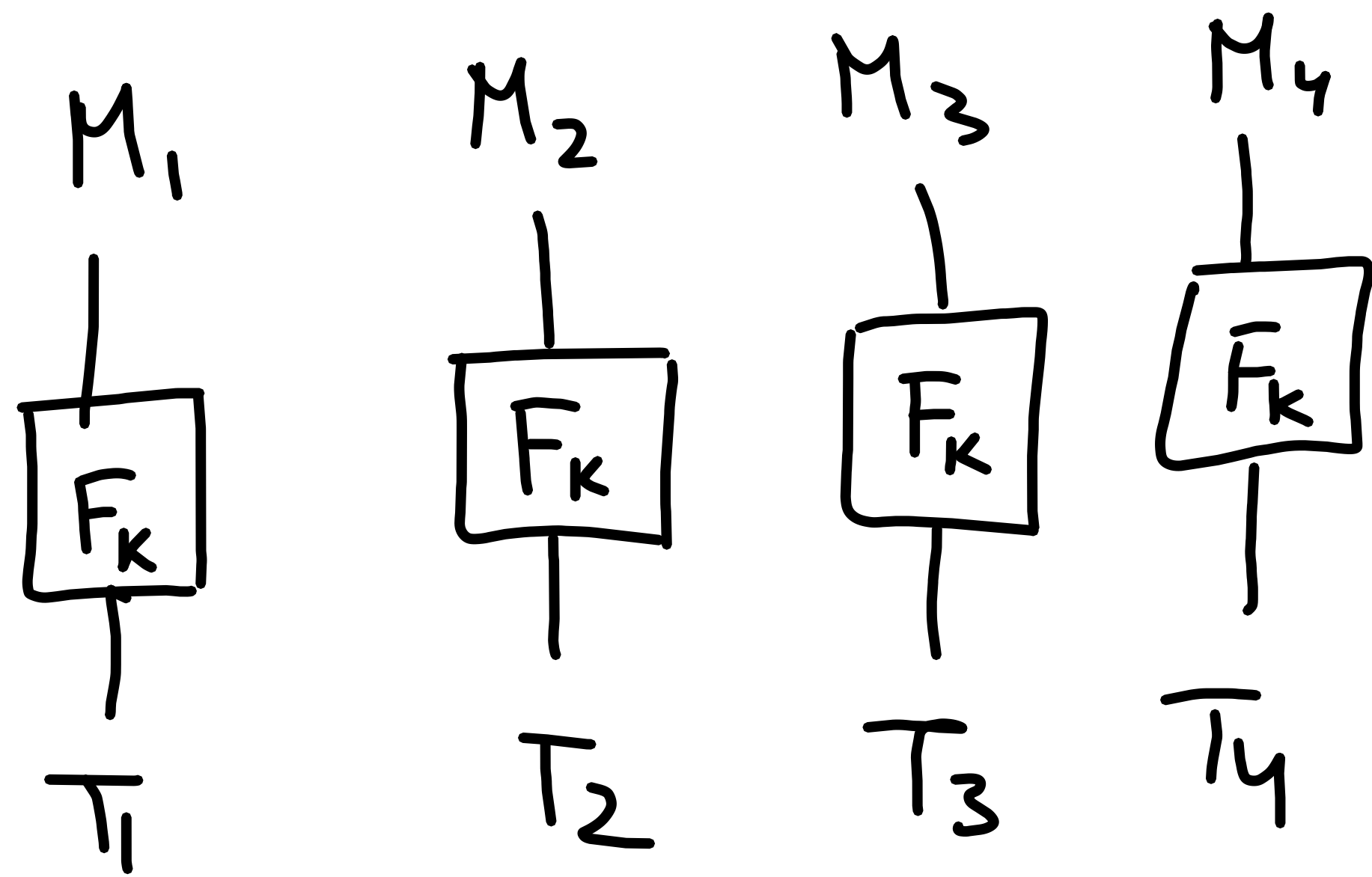
A



{ Valid
forgery



MACs for Variable length Messages



$$TG(k, M_1 || M_2 || M_3 || M_4) = (T_1 || T_2 || T_3 || T_4)$$

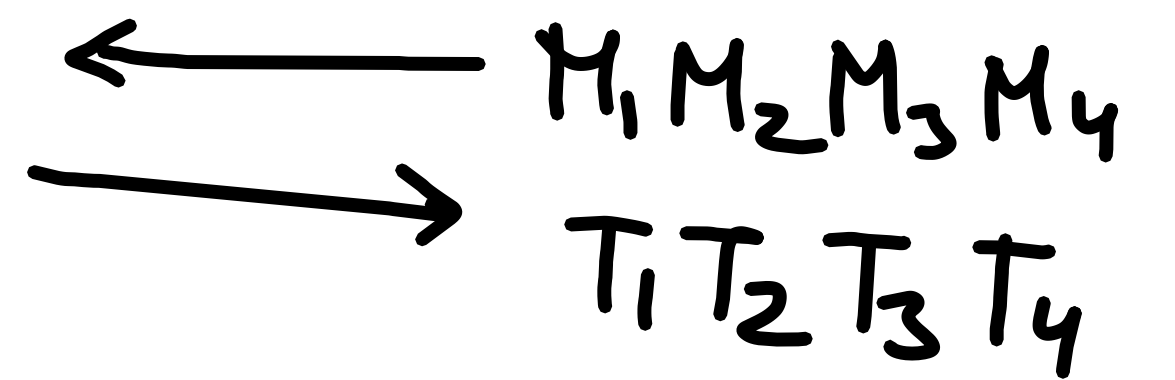
Is it EUF-CMA Secure?

No

(Re-arrange the blocks)

CR

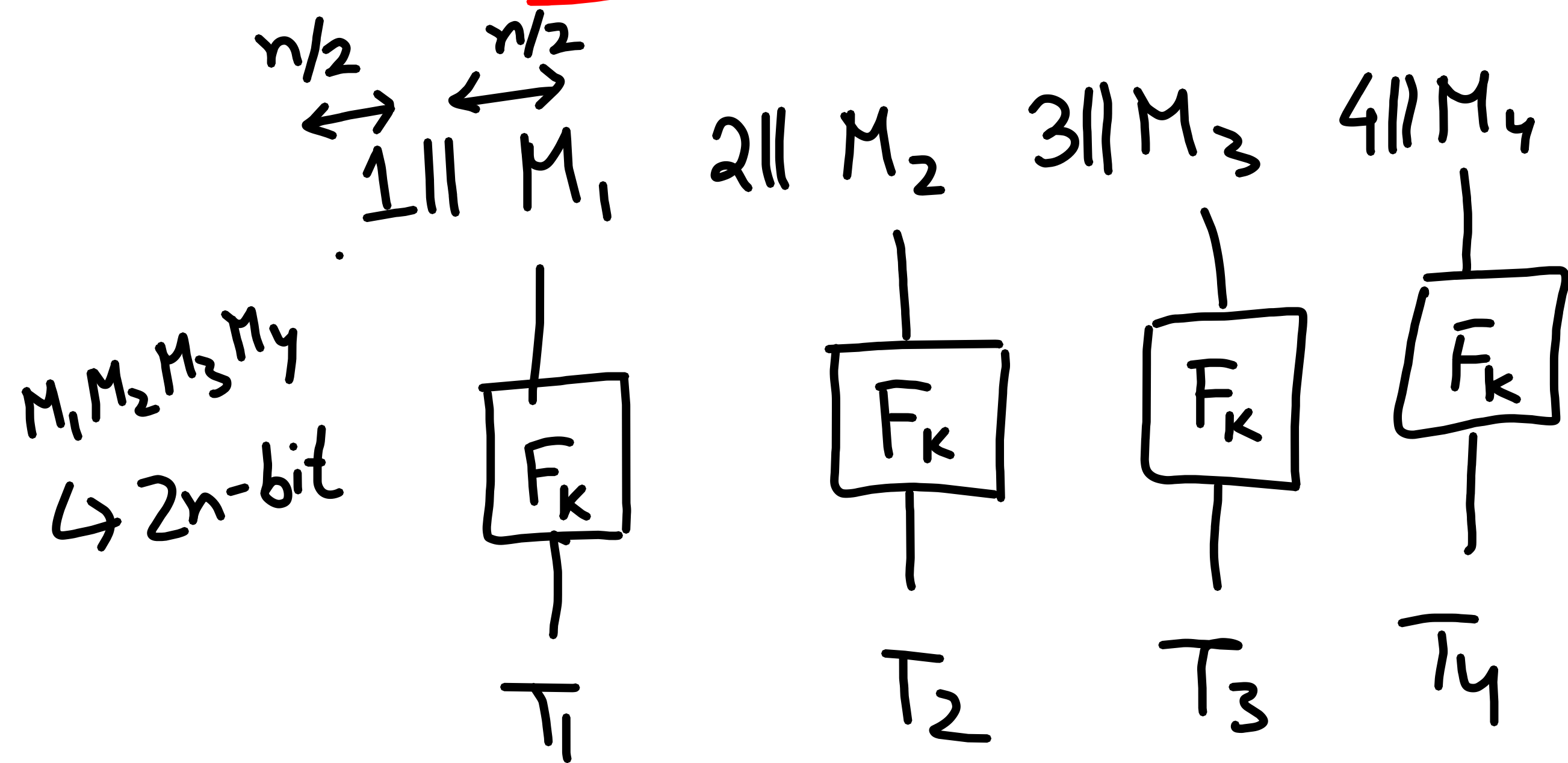
A



{ Valid
forgery

$(M_1 M_3 M_2 M_4, T_1 T_3 T_2 T_4)$

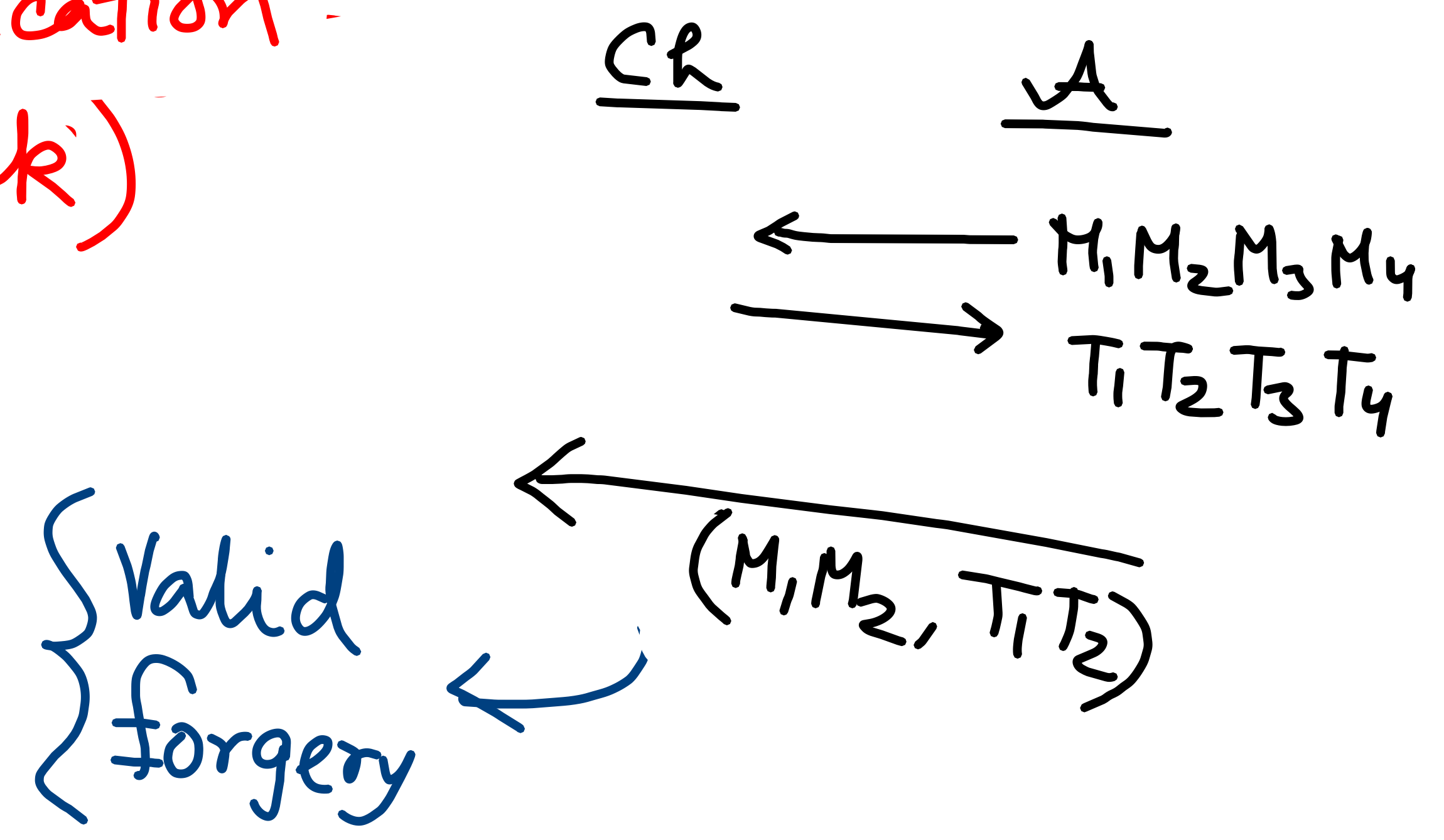
MACs for Variable length Messages



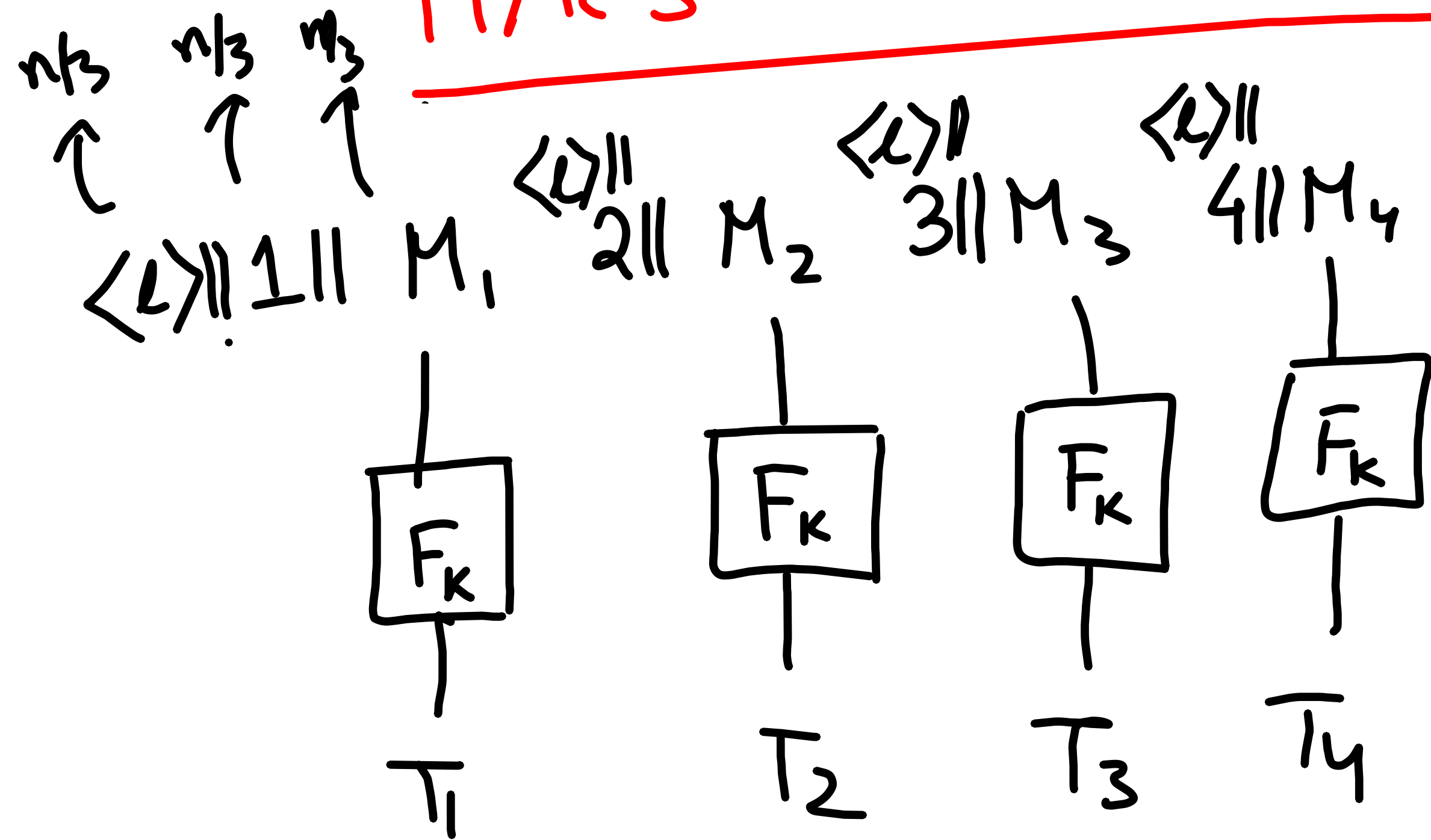
$M_1 M_2 M_3 M_4$
 $\hookrightarrow 2n$ -bit

$$\begin{aligned}
 &TG(k, M_1 || M_2 || M_3 || M_4) \\
 &= (T_1 || T_2 || T_3 || T_4)
 \end{aligned}$$

Is it EUF-CMA Secure?
 NO
 (Truncation Attack)



MACs for Variable length Messages

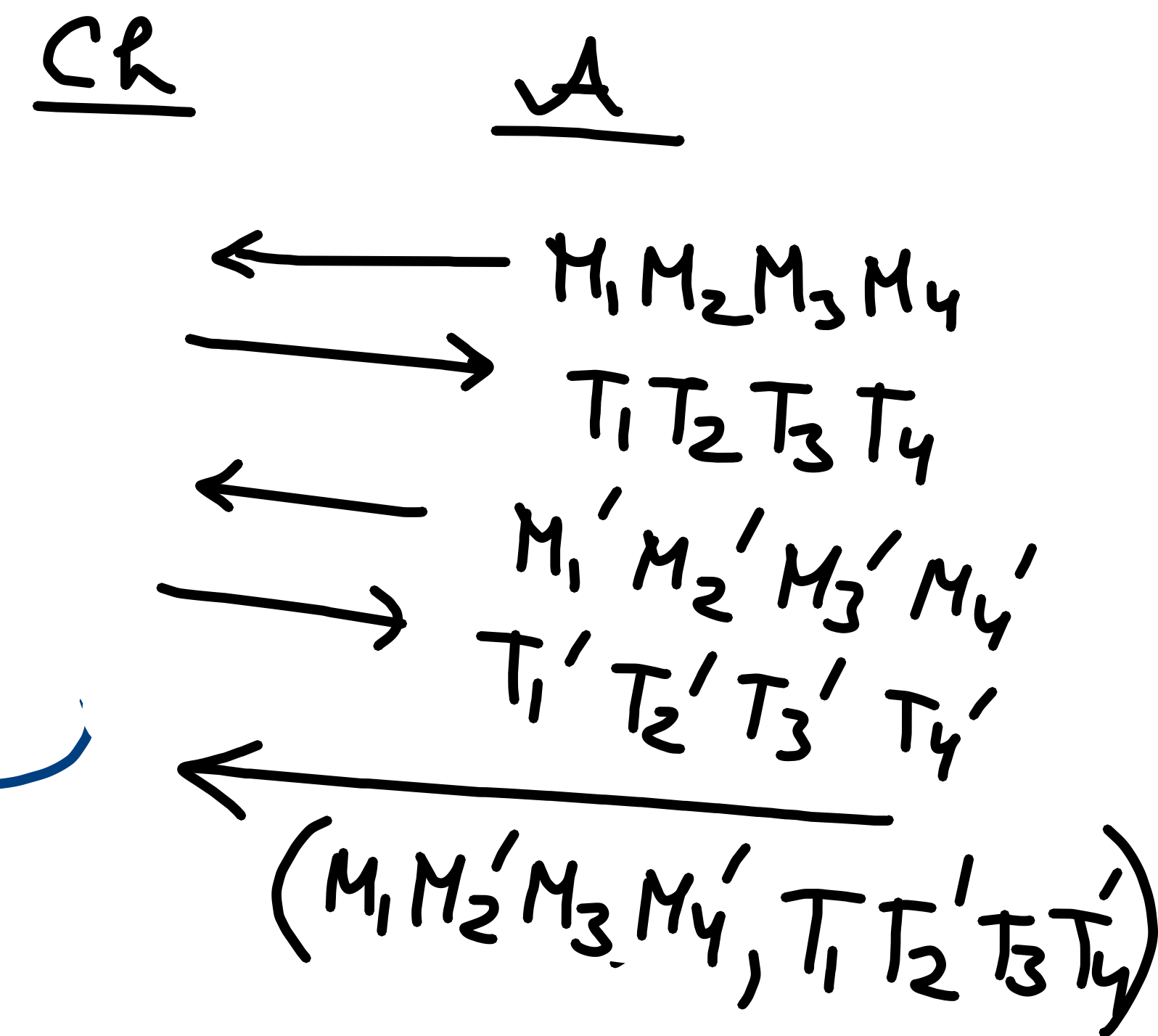


$$TG(k, M_1 || M_2 || M_3 || M_4) = (T_1 || T_2 || T_3 || T_4)$$

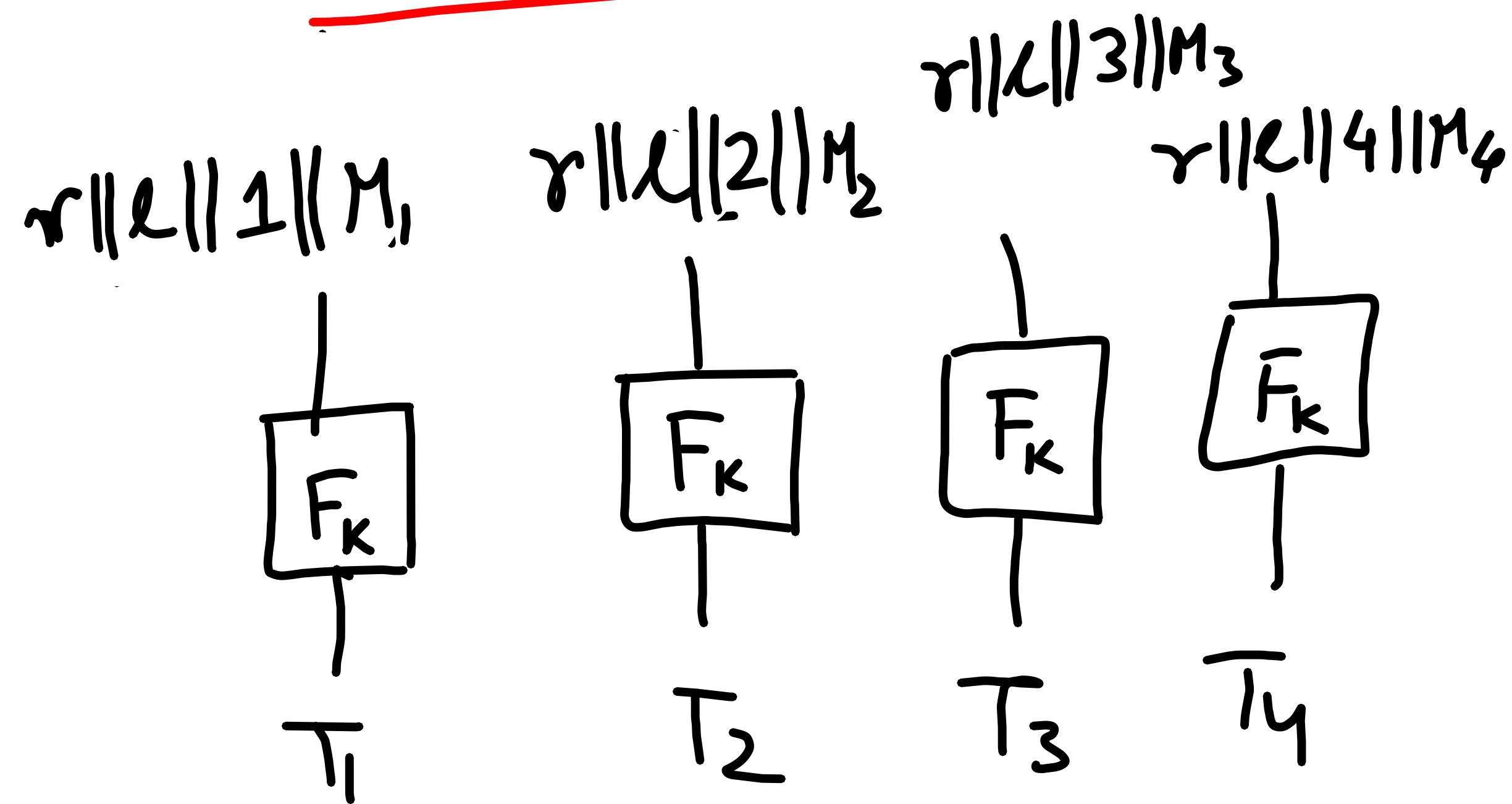
Is it EUF-CMA Secure?

NO
(Mix-&-Match Attack)

{ Valid forgery



MACs for Variable length Messages



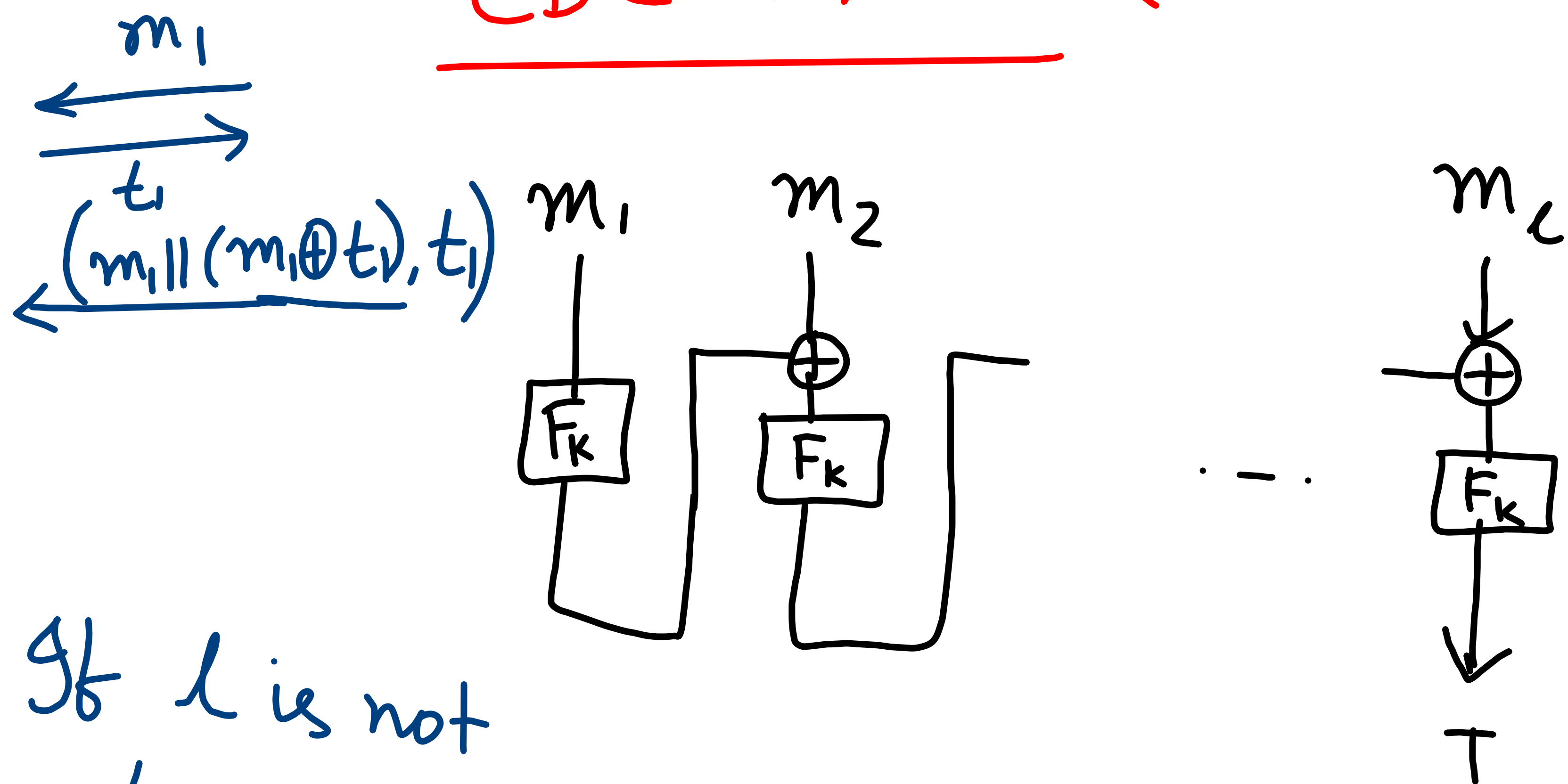
Is it EUF-CMA Secure?

Yes \Rightarrow

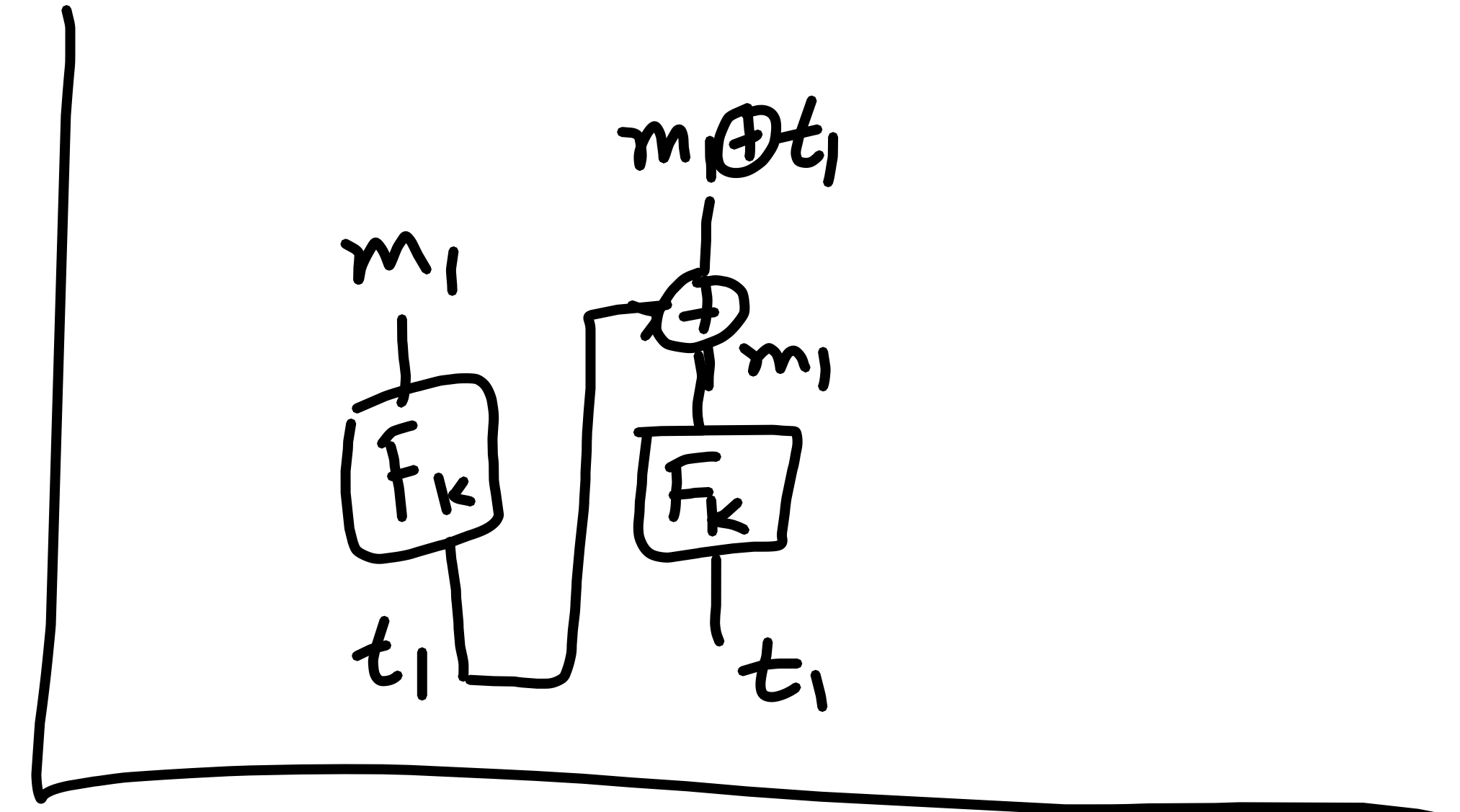
$$\frac{q^2}{2^{n/4}}$$

$$TG(k, M_1 || M_2 || M_3 || M_4) = (r || T_1 || T_2 || T_3 || T_4)$$

CBC-MAC (Fix length)



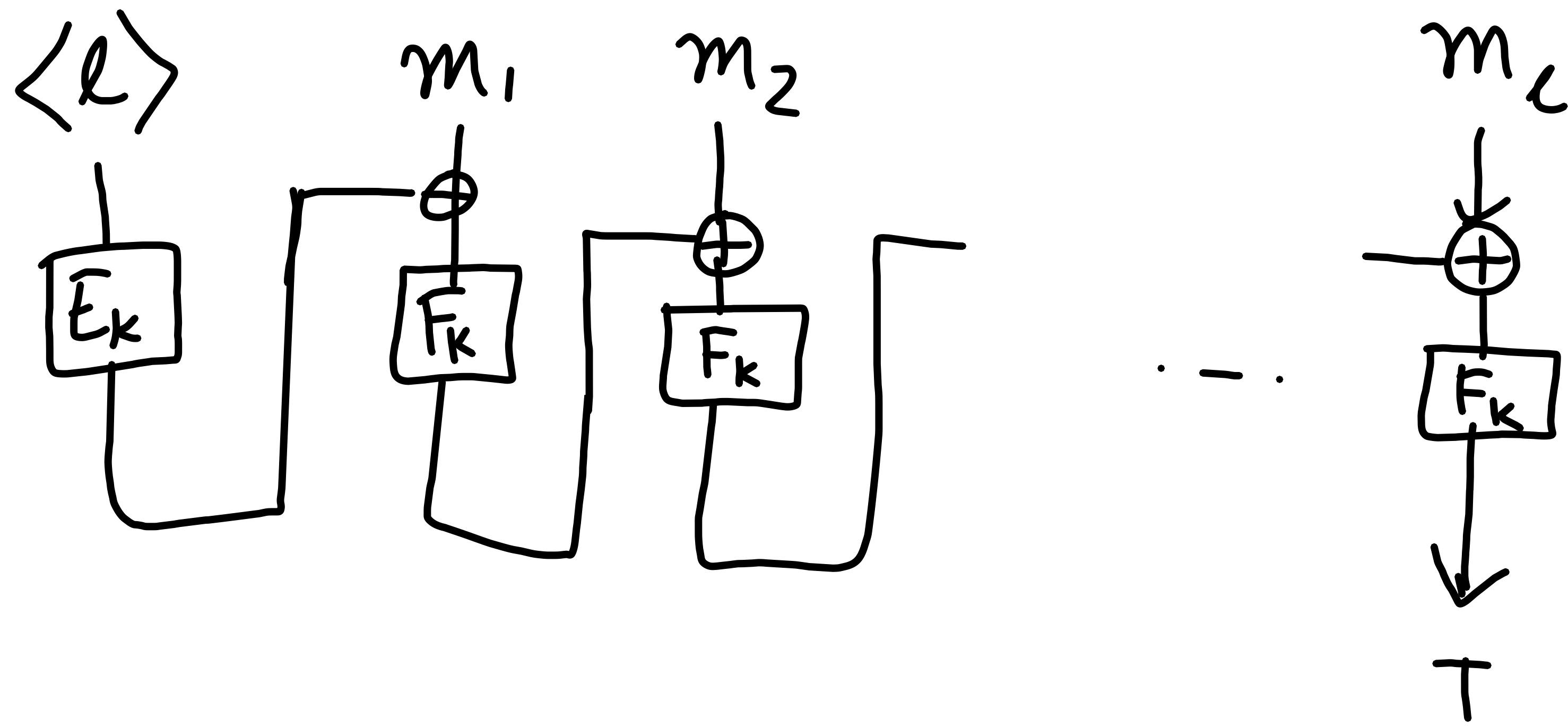
If l is not fixed \rightarrow Attack



$l \rightarrow$ fixed

Secure

CBC-MAC (Variable length)



$l \rightarrow \text{fixed}$

Secure

Recap (Last class)

Counter Mode

$$\Pr[\text{BAD}] \leq \sum_{i=1}^{q(n)} \frac{l_i \cdot l}{2^n} \Rightarrow \leq \sum_{i=1}^{q(n)} \frac{l_i + l}{2^n} = \frac{2 \cdot l_{\max}}{2^n}$$

$$\hookrightarrow \Pr[\text{PrivK}_{\text{CTR}}^{\text{ind-cpa}}(\mathcal{A}) = 1] \leq \frac{q(n) \cdot l_{\max}^2}{2^n} + \frac{1}{2}$$

$$\Downarrow \\ \frac{2 \cdot q(n) \cdot l_{\max}}{2^n} + \frac{1}{2}$$